Math 141b Problem Set 7

Due April 9, 2019

1. Recall from class that there is partial computable $\phi : \mathbb{N}^2 \to \mathbb{N}$ that enumerates all partial computable functions. Show that 'partial' is critical here; that is, show that there is no total computable $\phi : \mathbb{N}^2 \to \mathbb{N}$ such that, for every computable $f : \mathbb{N} \to \mathbb{N}$, there is $e \in \mathbb{N}$ such that for all $n \in \mathbb{N}$,

$$\phi(e, n) = f(n)$$

2. We saw how to build finite sets diagonal indiscernibles from a strengthening of (finite) Ramsey's Theorem. Now use Ramsey's Theorem to find finite sets of regular indiscernibles: Let Γ be a finite collection of formulas (in the language of arithmetic). Show that (in $\mathbb{N}$ or any model of PA) for every $n \in \mathbb{N}$, there are $x_1, \dots x_n$ that are Γ-indiscernible, which means, for every $\phi(y_1, \dots, y_k) \in \Gamma$ and $i_1 < \cdots < i_k \le n$ and $j_1 < \cdots < j_k \le n$, we have

$$\phi(x_{i_1}, \dots, x_{i_k}) \iff \phi(x_{j_1}, \dots, x_{j_k})$$

3. Finite is boring, so let's strengthen the previous problem to the infinite using the compactness theorem: Show that there is a model M of PA and $\{x_n : n \in \mathbb{N}\} \subset M$ such that, for any formula $\phi(y_1, \dots, y_k)$ in the language of arithmetic and $i_1 < \cdots < i_k \in \mathbb{N}$ and $j_1 < \cdots < j_k \in \mathbb{N}$, we have

$$\phi(x_{i_1}, \dots, x_{i_k}) \iff \phi(x_{j_1}, \dots, x_{j_k})$$