Math 141b Problem Set 2

Due Feb. 12, 2019

For simplicity, we will assume all our language τ are at most countable.

1. Characterize the models of T_L as we did with T_S. The following outline might be helpful:
   (a) $M \vDash T_L$ consists of a copy of $\mathbb{N}$ and an *ordered* set of $\mathbb{Z}$-chains
   (b) The ordering of these $\mathbb{Z}$-chains characterizes the models up to isomorphism
   (c) For each linear order I, find a model of T_L whose $\mathbb{Z}$-chains are ordered like I
2. Show that addition is not definable in $\mathbb{N}_L$
3. Show that an eventually definable set $A \subset \mathbb{N}$ is definable in Presburger arithmetic $\mathbb{N}_P$.
4. Work in the language consisting of a single binary relation $<$ (and equality). Let T be the following theory (often called 'dense linear orders without endpoints'):
   (a) Antireflexivity: $\forall x \neg(x < x)$
   (b) Antisymmetry: $\forall x, y(x < y \rightarrow y \not< x)$
   (c) Transitivity: $\forall x, y, z(x < y \wedge y < z \rightarrow x < z)$
   (d) Trichotomy: $\forall x, y(x < y \vee x = y \vee x > y)$
   (e) Dense: $\forall x, y(x < y \rightarrow \exists z(x < z < y))$
   (f) No Endpoints: $\forall x \exists y, z(y < x < z)$

   $(\mathbb{Q}, <)$ and $(\mathbb{R}, <)$ are the most familiar models of this theory. Show T has quantifier elimination.