

Partial Differential Equations

General Problem: We will have a partial differential equation in $f(x, t)$.

- x represents position and will be bounded in $[0, \pi]$.
- t represents time and will be ≥ 0 .
- Initial conditions will be of the form $f(x, 0) = g(x)$ for x in $[0, \pi]$. We will always assume that g is odd and 2π -periodic.
- D is the linear operator that takes a derivative with respect to x and only D^2 will appear.

Example:

$$f_t = 5f_{xx}$$

with $f(x, 0) = \max(\cos x, 0)$.

General Outline:

1. Write the equation in terms of D^2 .
2. Assume that $f(x, t)$ is an eigenvalue for the operator made up of all the D^2 's (using sin only!).
3. Solve the resulting ordinary differential equation with respect to t .
4. Combine the corresponding solutions and take their "span" (the sum of them).
5. Use Fourier series to solve for the initial values

1. Solve $f_t = -f_{xx}$ given $f(x, 0) = \sin x$ for x in $[0, 2\pi]$.

2. Solve

$$u_{tt} = u_{xxxx}$$

for x in $[0, \pi]$ given boundary conditions

$$\begin{aligned}u(x, 0) &= x \\u_t(x, 0) &= 3x\end{aligned}$$

3. Find $f(x, t)$ such that

$$f_t = -f_{xx} + \sin 7t$$

on $[0, \pi]$ given $f(x, 0) = x$.