Least Squares and Determinants

- Problems 1-2 cover the best-fit problems
- Problems 4-7 cover determinants

Let $A\vec{x} = \vec{b}$ be a (probably inconsistent) system of equations (where A probably has many more columns than rows)

1. We find the least squares solution to $A\vec{x} = \vec{b}$ by
2. If $\ker A = \{\vec{0}\}$, the formula for the least squares solution is

1. I made up this data about attendance in my class and temperature a few classes ago:

Temperature (T)	Students attending (S)
40	23
38	24
6	15
50	22
60	23

Find the best approximation ("least squares") for a linear function $f(T) = eT + f$ to approximate this data.

.

2. Find the best approximation of a quadratic function $ax^2 + bx + c$ to approximate the points $(-1, 2)$, $(0, 1)$, $(1, 3)$, and $(2, -1)$.

3. What is the determinant of $\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}$?

Let π be a rearrangement of $\{1, \ldots, n\}$ and A be an $n \times n$ matrix with entries a_{ij}.

(a) Prodπ =

(b) $|\pi|$ =

An upcrossing is

(c) The determinant is

$$det(A) =$$

4. What is the determinant of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$?

5. Find $\begin{bmatrix} 6 & 0 & 1 & 0 & 0 \\ 9 & 3 & 2 & 3 & 7 \\ 8 & 0 & 3 & 2 & 9 \\ 0 & 0 & 4 & 0 & 0 \\ 5 & 0 & 5 & 0 & 1 \end{bmatrix}$.

6. Find the determinant of an $n \times n$ triangular matrix (entry in the i row and j column is 0 when $j < i$)

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{nn} \end{bmatrix}$$

7. Let A be the matrix $\begin{bmatrix} 1 & -1 & 3 \\ 4 & 5 & 7 \\ 0 & 2 & 0 \end{bmatrix}$.

(a) Find the determinant by expanding along the first row.

(b) Find the determinant by expanding along the bottom row.