

## Introduction to Linear Systems

1. Solve the following system of equations related to the Google Page Rank example.

$$\begin{aligned}x_1 &= \\x_2 &= \frac{1}{3}x_1 + \frac{1}{2}x_3 + \frac{1}{2}x_4 \\x_3 &= \frac{1}{3}x_1 \\x_4 &= \frac{1}{3}x_1 + \frac{1}{2}x_3 + x_5 \\x_5 &= x_2\end{aligned}$$

Which website would you rank first? What could you do to make the third website ranked more highly?

2. Solve the following system of equations

$$\begin{aligned}x + 2y + 3z &= 1 \\2x + 4y + 7z &= 2 \\3x + 7y + 11z &= 8\end{aligned}$$

3. Solve the following system of equations

$$\begin{array}{rccccrcrcl} & & & -2x_3 & + & 8x_4 & - & 3x_5 & = & -4 \\ & & & 3x_3 & - & 12x_4 & & & = & 15 \\ x_1 & + & 2x_2 & - & x_3 & + & 7x_4 & & = & 2 \end{array}$$

4. Write the above systems in matrix form.

5. Consider the linear system

$$\begin{array}{rcl} x & + & 2y = 3 \\ x & + & 2y = 7 \end{array}$$

(a) Write this system in matrix form.

(b) Just looking at this system, what can you say about the number of solutions?

(c) What happens if you solve using the method of elimination?

Consider the linear system below. For what values of  $k$  is the system inconsistent? For what values of  $k$  does it have exactly one solution? For what values of  $k$  does it have more than one solution? (In this last case, how many solutions does it have?)

$$\begin{array}{rcl} 2x & + & 2y & + & kz & = & 3 \\ kx & + & ky & + & 8z & = & k + 2 \end{array}$$