MATH 255: CLASSIFICATION THEORY FOR TAME ABSTRACT ELEMENTARY CLASSES

Instructor: Will Boney
Class Location: SC 105
Class Time: TR 10:00-11:30am
Textbook: None, but see references
Instructor's Office: Science Center 238
Course Website: `http://www.math.harvard.edu/~wboney/fall17`
(Note the electronic version of this document on the website has some useful embedded hyperlinks)
Office Hours: officially by appointment, but unofficially I'm normally in my office most days 9-5 and you're welcome to stop by

1. Course Description

Classification theory in first-order model theory is a powerful tool that *classifies* first-order theories along certain dividing lines: stability, simplicity, etc. (see forkinganddividing.com for a great pictorial representation). Theories on the "well-behaved" side of these lines come with a notion of independence or non-forking similar to linear independence in vector fields or algebraic independence in algebraically closed fields (of course, how similar they are depends on the dividing line).

The development of classification uses the compactness theorem heavily, so doesn't apply to nonelementary contexts, such as locally finite groups or Zilber's pseudoexponentiation. Nonetheless, classification theory can be done in a restricted way (so far) in many nonelementary contexts. One property that has found much fruitful use in the endeavor is the notion of *tameness*, first isolated by Grossberg and VanDieren. This is a weakening of compactness that gives a local criteria for when to types (appropriately defined) are equal.

Starting from the definition of Abstract Elementary Classes (AECs), we will develop nonforking and classification theory for tame AECs. We will start with proving theorems of Shelah, Grossberg-VanDieren, and Boney about categoricity transfer and tameness. This will also serve as a way to introduce many of the basic concepts of tame AECs: Galois types, EM models, etc. From there, based on class interest and time, we will explore some of the following: set-theoretic applications to AECs, development of nonforking in tame AECs, and examples of tame AECs.

1.1. References.

There are nice hidden hyperlinks in the online version.

(1) John Baldwin, **Categoricity**. This available through the author's website (errata are also available).
(2) Rami Grossberg, *Classification Theory for Abstract Elementary Classes*. This available through the author's website.
(3) Will Boney and Sebastien Vasey, *A Survey on Tame Abstract Elementary Classes*. This available through the author's website.
(4) Saharon Shelah, **Classification Theory for Abstract Elementary Classes**. Individual chapters are available through the author's website, where the introduction is [ShE53].

You don't have to get these, although you might find them useful in getting additional perspective on the areas we cover.

2. Requirements

This course will cover all the basics of AECs, but will assume a basic familiarity with concepts coming from set and model theory. Depending on undergraduate enrollment, assessment will take the form of presentations and/or problem sets.