“WHY IS $\{f \mid f$ is good$\}$ A SET?”

SEP. 13

During the proof of Transfinite Recursion today, Daniel pointed out that I hadn't justified that the collection of all good functions was a set. Since we're only allowed to talk about sets, we need this justification. The proof turns out to make important use of the Axiom Schema of Replacement.

We showed that, for each $D \subset X$, there is at most one good function with domain D. Define the set

$$Y := \{D \subset X \mid \exists f \text{ that is a good function with domain } D\}$$

We discussed the definition of being good as a first-order formula in class. By the uniqueness, the association of $D \in Y$ and the good function f with domain D is a definition of a function. So, by the Axiom Schema of Replacement, $\{f \mid f$ is a good function with domain $D\}$ is a set; this is just a more complicated description of $\{f \mid f$ is good$\}$.

Then the proof can proceed as in class.