MATH 145A: SET THEORY

Instructor: Will Boney
Class Location: SC 411
Class Time: TR 1:00pm-2:30pm
Textbook: Peter Koellner, *Set Theory: The Independence Phenomenon*
Instructor's Office: Science Center 231
Course Website: `http://www.math.harvard.edu/~wboney/fall16`
(Note the electronic version of this document on the website has some useful embedded hyperlinks)

Monday	12:30-1:30pm
Thursday	3:00-4:00pm

1. Course Description

Set theory concerns itself with the basic foundations of math, on which all other mathematics is based (at least according to set theorists). Set theory beings with a set of axioms (ZFC) in which all of mathematics can be formalized, and then asks what lies beyond these axioms. The course is roughly divided into thirds.

The first third of the course covers the basics of set theory. We will begin by exploring the axioms of ZFC, and then go on to develop a theory of cardinals and ordinals. These form the basis of the set-theoretic world.

The second third will introduce two topics. Descriptive set theory classifies sets (especially sets of reals) according to how complicated it is to define them. This gives us a good framework to examine the phenomena of non-measurable sets, and how many sets we can require to be measurable. Trees (in the graph-theoretic sense) are a fertile ground for various properties in combinatorial set theory, and we will discuss their application to Suslin's Hypothesis.

The final third brings us to the topic independence by introducing G odel's Constructible Universe L. L is the minimal model of set theory in which every set is definable (in the appropriate sense). This universe provides a useful model for many axioms beyond ZFC, such as the continuum hypothesis. To get models for other axioms, we must use forcing, which we will briefly outline at the end of the course (time permitting).

1.1. References.

In addition to the *Set Theory*, there are a few other standard set theory texts that are useful references (and good proof that set theorist are not creative with book names).

(1) Kenneth Kunen, *Set Theory: An introduction to independence proofs*. This available through Harvard's library.

(2) Thomas Jech, *Set Theory*. This is available (on Harvard's network) through Springer's website. Be careful: this is a better reference than introduction and contains much more material than we will cover.

You don't have to get these, although you might find them useful in getting additional perspective on the areas we cover.

2. Requirements

2.1. **Homework.** Every week, we will have homework due on Tuesday in class. Homework will be assigned in class and posted to the course website. At the latest, the assignments will be posted when you turn in the previous ones. Late homework will not be accepted.

I encourage you to collaborate with classmates. We will be covering some hard material, and often quickly. Discussing these concepts with other students is one of the best ways to get to the heart of the matter. The new Math Night is an excellent venue for this. On the other hand, homework exists in order to give you practice with this material and help you gauge your understanding of it. Thus, it is important to be sure that you are an active collaborator. You're the only true judge of this, but a good check for homework is to work on problems together, but write them up independently.

As for doing the homework, the work that goes into the answer, rather than just the answer, is the most important part. Thus, all problems require justification. Failure to give justification will result in no credit for the problem. Not providing enough justification will also result in lost points.

2.2. **Exams.** There will be two exams: a midterm on Tuesday, October 11 during class and a final on Monday, December 12 2:00pm-5:00pm. The midterm will cover the material up till that point. The final will be cumulative, but will focus more on the material covered since the midterm.

2.3. **Percentage Breakdown.**

Homework	60%
Midterm	20%
Final	20%

The maximum possible grade cutoffs will be 90, 80, 70, and 60 for A, B, C, and D, respectively. Based on the scores at the end of the course, these might be lowered, but will not be increased.

2.4. **Policies.** Several other general policies (classroom courtesy, academic integrity, etc.) are listed on website in the appropriate section (this is one of those hidden hyperlinks in the online version).