MATH 145A HOMEWORK 8

DUE NOV. 15

(1) A tree T is κ-embeddable iff there is $f : T \to \kappa$ such that if $x <_T y \in T$, then $f(x) \neq f(y)$ (this is the definition from class, but different than the definition in the book). Show that a tree is κ-special iff it is κ-embeddable.
(2) Let T be a κ^+-tree that is κ-embeddable. Show that it doesn't have a cofinal branch, but does have a κ^+-sized antichain.
(3) Exercise 4.6
(4) Show how to modify the proof that "there is a Suslin tree implies there is a normal Suslin tree" to get a normal Suslin tree with the additional property that every node has *inftinitely* many successors. (Hint: Let T^5 be all nodes of T^4 on limit levels)
(5) Recall the definition of a club from Homework 6. Suppose we have the following:
    - λ be an uncountable, regular cardinal;
    - the sequence $\{X_\alpha : \alpha < \lambda\}$ of sets is increasing ($\alpha < \beta$ implies $X_\alpha \subsetneq X_\beta$), continuous (limit α implies $X_\alpha = \cup_{\beta<\alpha} X_\beta$), and $|X_\alpha| < \lambda$;
    - $X = \cup_{\alpha<\lambda} X_\alpha$; and
    - $f : X \to X$ and $g : X^2 \to X$ are functions.

    Show that
    $$\{\alpha < \lambda : f"X_\alpha \subset X_\alpha \text{ and } g"X_\alpha^2 \subset X_\alpha\}$$
    is club in λ.