MATH 145A HOMEWORK 7

DUE NOV. 8

(0) Be sure to vote if you're able (0 points).
(1) For every $\alpha < \omega_1$, find $A_\alpha \subset \mathbb{Q}$ such that $(A_\alpha, <_{\mathbb{Q}})$ is a well-ordering and $otp(A_\alpha, <_{\mathbb{Q}}) = \alpha$ ($<_{\mathbb{Q}}$ is the normal ordering on the rationals).
(2) In the proof of Corollary 4.2 from the book, show that f^* is an isomorphism.
(3) Exercise 4.1
(4) Let $(T, <_T)$ be a κ-tree such that for every limit $\alpha < \kappa$ and $x, y \in T_\alpha$, if $pred_T(x) = pred_T(y)$, then $x = y$.
   (a) Show that there is an injection $f : T \to {}^{<\kappa}\kappa$ such that, for all $x, y \in T$,
   $$x <_T y \iff f(x) \subset f(y)$$
   (b) What do branches in T correspond to in terms of ${}^{<\kappa}\kappa$?
   (c) Show that $({}^{<\kappa}\kappa, \subset)$ is not a κ-tree.

   This tells you that every κ-tree "can be seen as" a subtree of ${}^{<\kappa}\kappa$ with an additional property. (c) should give you a hint as to what the additional property is.
(5) Show that the $\aleph_1$-Aronszajn tree constructed in class is $\aleph_0$-special and satisfies the following conditions from normality:
   (a) $|T_0| = 1$
   (b) For all $\alpha < \beta < \omega_1$, if $x \in T_\alpha$, then there is $y \in T_\beta$ such that $x <_T y$
   (c) For all $\alpha < \omega_1$, if $x \in T_\alpha$, then there are $y_1 \neq y_2 \in T_{\alpha+1}$ such that $x <_T y_1$ and $x <_T y_2$.

   (Be careful: we used a slightly different construction than in the book)

Exercise 4.7 is very interesting, but (as the book indicates) hard. You can do Exercise 4.7 in place of any three of the above problems (except for voting).