MATH 145A HOMEWORK 4

DUE OCT. 4

(1) Exercise 2.3
(2) Suppose that $\langle X_\alpha \mid \alpha < \beta \rangle$ is a sequence of sets with $X_\alpha \subset X_{\alpha+1}$ and, if $\delta < \beta$ is a limit ordinal, then $X_\delta = \cup_{\alpha<\delta} X_\alpha$. Show that, for $\alpha < \gamma < \beta$, $X_\alpha \subset X_\gamma$.
(3) Exercise 2.4
(4) Exercise 2.5
(5) Exercise 2.8
(6) (a) Suppose that $\mathrm{rank}(x) = \alpha$. What is $\mathrm{rank}\left(\mathcal{P}(x)\right)$?
    (b) Suppose that $\mathrm{rank}(x) = \alpha$, $\mathrm{rank}(x) = \beta$ with $\alpha \leq \beta$. What is $\mathrm{rank}(\{x, y\})$?
(7) For each of the following, find pairs of cardinals such that the cardinal and ordinal version of the operations differ.
    (a) Addition
    (b) Multiplication
    (c) Exponentiation
(8) My first set theory professor was obsessed with the fact $\kappa + \lambda = \kappa\lambda = \max\{\kappa, \lambda\}$, called it "the collapse of cardinal arithmetic," and gave us tons of exercises to compute some cardinal arithmetic. I will only give you one (at least on this homework). Set $\mathfrak{c} = 2^{\aleph_0}$. Show that

$$\mathfrak{c}^{\aleph_0} + \aleph_0 \cdot \aleph_0 = \mathfrak{c}$$