MATH 145A HOMEWORK 10

DUE DEC. 1

(1) Exercise 6.6
(2) Exercise 6.7 (repeat from last time, can redo if you want)
(3) Exercise 6.8
(4) The following problem counts for two and will set up our discussion of forcing on Thursday. (However, you still have to do all problems)

Let $Add(\omega, \kappa)$ be the collection of partial functions $p : \kappa \times \omega \to 2$ such that $|p| < \omega$ and order it by reverse inclusion. We say that $G \subset (Add(\omega, \kappa), \supset)$ is a filter iff

- $\forall p \in G \forall q \in Add(\omega, \kappa)\, (q \subset p \to q \in G)$
- $\forall p, q \in G \exists r \in G\, (p \subset r \wedge q \subset r)$

$D \subset Add(\omega, \kappa)$ is called dense iff for every $p \in Add(\omega, \kappa)$, there is $q \in D$ such that $p \subset q$. We say G meets D iff $G \cap D \neq \emptyset$. Show the following:

(a) For each $n < \omega$ and $\alpha < \kappa$, set $D_{n,\alpha} = \{p : (\alpha, n) \in \text{dom } p\}$. Show that each $D_{n,\alpha}$ is dense and, if G meets each $D_{n,\alpha}$, then $\cup G$ is a function from $\kappa \times \omega \to 2$. In this case, set $X^G_\alpha = \{n < \omega : \cup G(\alpha, n) = 1\}$.

(b) For $\alpha \neq \beta < \kappa$, set

$$Diff_{\alpha,\beta} = \{p : \exists n < \omega, p(\alpha, n) \neq p(\beta, n)\}$$

(Whenever we write $p(x)$ for p a partial function, we are also saying that $x \in \text{dom } p$)

If G meets each $D_{n,\alpha}$ and $Diff_{\alpha,\beta}$, then show all X^G_α are distinct.

(c) If $A \subset \omega$ is non empty and $\alpha < \kappa$, set

$$D^A_\alpha = \{p : \exists n \in A, p(\alpha, n) = 0\}$$
$$D^\emptyset_\alpha = \{p : \exists n < \omega, p(\alpha, n) = 1\}$$

Let $X \subset \mathcal{P}(\omega)$ and suppose that G meets each $D_{n,\alpha}$, each $Diff_{\alpha,\beta}$, and each D^A_α for $A \in X$ and $\alpha \neq \beta < \kappa$. Use the above to define κ many distinct subsets of ω that are not in X.

(d) Is it possible to have a filter G that meets each $D_{n,\alpha}$, each $Diff_{\alpha,\beta}$, and each D^A_α for $A \subset \omega$?