MATH 145A HOMEWORK 1

DUE SEP. 6

These exercises are intended to get you familiar with doing things set theoretically. Remember, Math Night is Monday at Leverett House's dining hall starting at 8pm and is a great place to meet with classmates about the problems.

(1) Recall that we defined the ordered pair of x and y as $(x, y) := \{\{x\}, \{x, y\}\}$.
   (a) Prove that this definition works. That is, prove that $(x, y) = (a, b)$ iff $x = a$ and $y = b$.
   (b) For each $n \in \mathbb{N}$, give a definition for the ordered n-tuple $(x_1, \ldots, x_n)$.

(2) For each $n \in \mathbb{N}$, show that $n = \{m \in \mathbb{N} \mid m < n\}$.

(3) We went through the construction of the reals fairly quickly. Define addition, multiplication, subtraction, and division. Then, pick one of these and show it's well-defined.

(4) Take your favorite mathematical object (that we haven't discussed); if you don't have a favorite, take $\mathbb{Z}_6$. Write this mathematical object as a set theorist would, i. e., just using curly braces. You're allowed to use abbreviations and shorthand here, but don't forget to code all the structure (function, relations, etc.) on the object. Bonus points for interesting objects.

(5) Set $x = \{y \mid y \notin y\}$.
   (a) Is $x \in x$?
   (b) Conclude that there is a problem.