145A MIDTERM REVIEW

You should be able to do the following:

(1) State the definitions of the concepts we've defined (ordinals, cardinals, their arithmetic, functions, equivalence relations, etc.)
(2) Identify the appropriate axiom of ZFC being used and show equivalent formulations of them
(3) State and prove the shorter named theorems
(4) Use Transfinite Recursion and Transfinite Induction
(5) Do computations with ordinal and cardinal arithmetic
(6) Find the cardinality of certain sets
(7) Find the cofinality of limit ordinals

Here are some practice problems:

(1) Prove the equivalence between the Axiom of Choice and it's equivalent statements.
(2) Prove Cantor's Theorem.
(3) Use the Schröder-Bernstein theorem to to show $\mathcal{P}(\omega)$ is the same size as $\mathbb{R}$.
(4) Make explicit use of Transfinite Recursion and Transfinite Induction in
    - The comparability theorem for well-orderings
    - The definitions of ordinal and cardinal arithmetic
(5) Show that, for any infinite cardinal, $\kappa \cdot \kappa = \kappa$ (with cardinal arithmetic).
(6) Show that ω^ω as ordinal exponentiation is countable, but ω^ω as cardinal exponentiation is uncountable.
(7) For limit ordinals α and β, find cf $\alpha + \beta$ in terms of cf α and cf β.