

FORCE FIELDS

MATH 21A

And other topics that aren't force fields

1. Match each gradient field to vector field. Yes, I know there are more items than gradient fields. Then, draw a few flow lines on each gradient field.

(a) $\mathbf{F}(x, y) = \langle 1, x \rangle$

(b) $\mathbf{F}(x, y) = \langle -y, x \rangle$

(c) $\mathbf{F}(x, y) = \langle y, x \rangle$

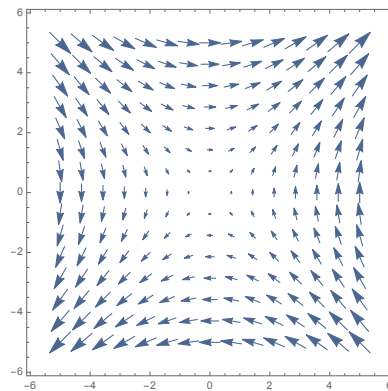
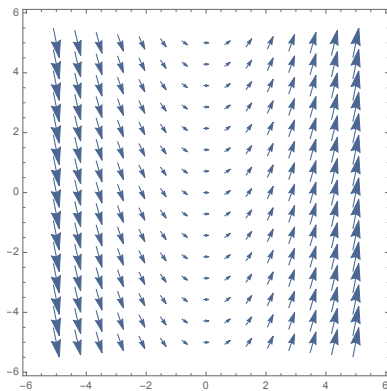
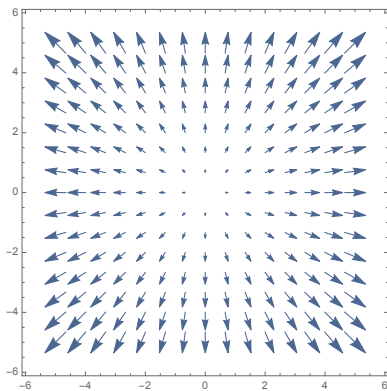
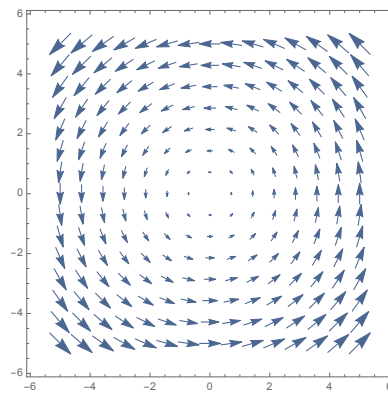
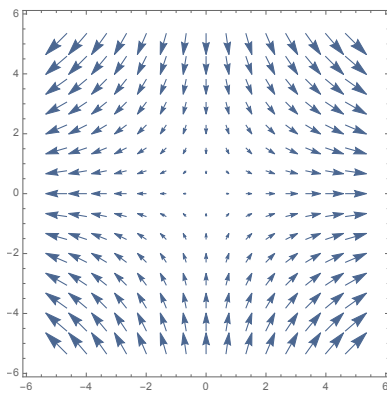
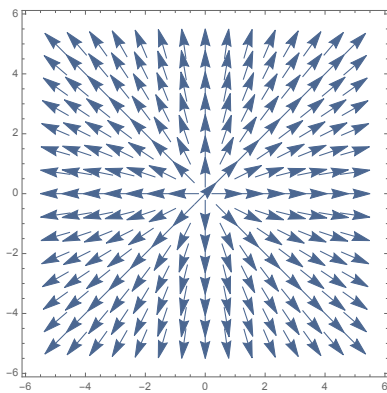
(d) $\mathbf{F}(x, y) = \langle 2x, -2y \rangle$

(e) ∇f , where $f(x, y) = x^2 + y^2$

(f) ∇f , where $f(x, y) = \sqrt{x^2 + y^2}$

(g) ∇f , where $f(x, y) = xy$

(h) ∇f , where $f(x, y) = x^2 - y^2$



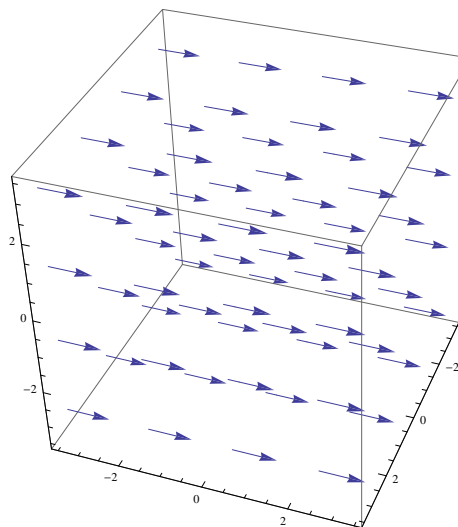
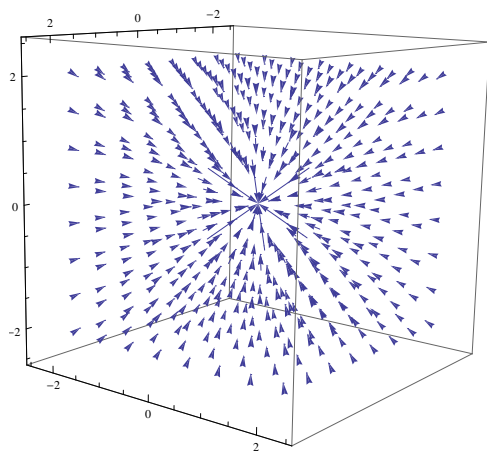
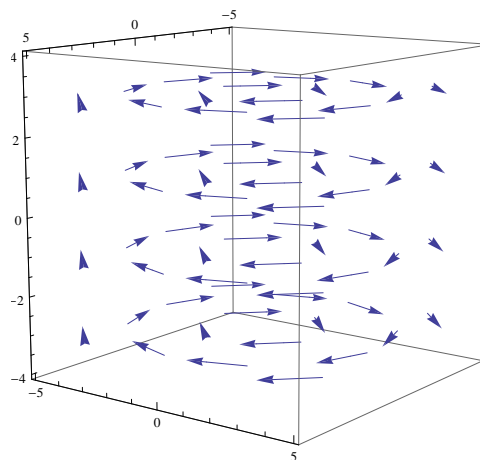
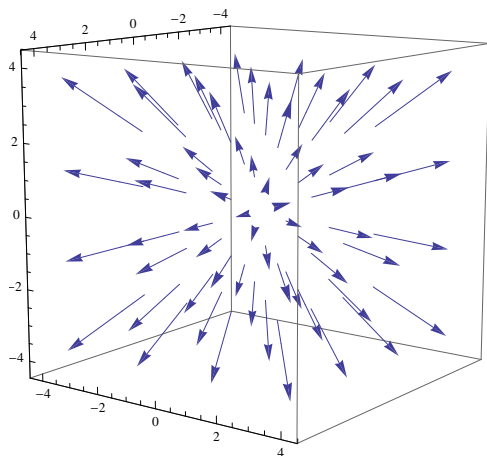
2. Here are some examples in three-dimensional space. The vector fields are

(a) $\mathbf{F} = \langle x, y, z \rangle$

(b) $\mathbf{F} = \langle 0, 1, 0 \rangle$

(c) $\mathbf{F} = \left\langle \frac{y}{\sqrt{x^2+y^2}}, -\frac{x}{\sqrt{x^2+y^2}}, 0 \right\rangle$

(d) $\mathbf{F} = \left\langle -\frac{x}{(x^2+y^2+z^2)^{3/2}}, -\frac{y}{(x^2+y^2+z^2)^{3/2}}, -\frac{z}{(x^2+y^2+z^2)^{3/2}} \right\rangle$



3. Determine if the following vector fields \mathbf{F} are gradient fields. If they are, find a potential, i. e., f so $\nabla f = \mathbf{F}$.

(a) $\langle y, x \rangle$

(b) $\langle x, y \rangle$

(c) $\langle x^2y, 2x \rangle$

(d) $\langle 2x \sin y, x^2 \cos y \rangle$

(e) $\langle 3x^2, x - 4y \rangle$

(f) $\langle 2y^2 + e^{x-y} + 1, 4xy - e^{x-y} + 2 \rangle$

Recall that

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt$$

If $\mathbf{F} = \langle P, Q \rangle$ and $\mathbf{r} = \langle x, y \rangle$, we sometimes write this as $\int_C P dx + Q dy$.

4. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is given by the vector function $\mathbf{r}(t)$ with the given bounds.

(a) $\mathbf{F}(x, y) = \langle xy, 3y^2 \rangle$, $\mathbf{r}(t) = \langle 11t^4, t^3 \rangle$ for $0 \leq t \leq 1$.

(b) $\mathbf{F}(x, y) = \langle -y, x \rangle$ and C is the clockwise path along the circle of radius 2 from $(2, 0)$ and $(-2, 0)$.

(c) $\mathbf{F}(x, y, z) = \langle x, y, -2z \rangle$, $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ for $0 \leq t \leq 1$.

(d) $\mathbf{F}(x, y, z) = \langle y, x, z^2 \rangle$ and C is the line segment connecting $(1, 1, 1)$ to $(2, -1, 0)$.

5. Pick two paths from $(1, 2)$ to $(3, 4)$. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ for both of them when \mathbf{F} is $\langle y, x \rangle$.

6. Do the above for $\mathbf{F} = \langle x^2y, 2x \rangle$.