# An integral so nice, we iterated it thrice

Math 21a

Or: I'm sad I didn't come up with that pun during double integration

In each of these problems, we focus on setting up the integral rather than evaluating it.

**1.** Set up the integral to find the volume of the solid bounded by $y = 0$, $x = 0$, $z = 0$, and $x + y + z = 1$.

**2.** Set up $\iiint_E z dV$, where $E$ is bounded by $x = y^2$, $x = z$, $z = 0$, and $x = 1$.

**3.** Express the integral $\iiint_E f(x,y,z)dV$ in multiple different ways, where $E$ is bounded by $y = x^2$, $z = 0$, and $y + 2z = 4$.

**4.** Set up the integral to find the mass of a solid bounded by $x = 4y^2 + 4z^2$ and $x = 4$ with density $\rho(x,y,z) = x + y + z$.

**5.** * Set up the integral to integrate $f(x, y, z)$ over the tetrahedron with vertices $(1, 0, 0)$, $(-1, 2, -1)$, $(3, 4, 0)$, and $(0, 1, 2)$.

When changing coordinate systems, we add an integrating/stretching factor as we do with polar:

| Coordinates | Factor |
|---|---|
| Spherical | $\rho^2 \sin\phi$ |
| Cylindrical | $r$ |

**6.** Set up

$$\iiint_E e^{\sqrt{x^2+y^2+z^2}} dV$$

, where $E$ is the solid enclosed by the sphere of radius 3 in the first octant.

**7.** Set up the integral to find the volume of the solid that is all points inside the cylinder $x^2 + y^2 = 1$, above the sphere $x^2 + y^2 + z^2 = 1$, and below the plane $z = 4$ in cartesian and cylindrical. Which looks easier to evaluate? Could you set this up in spherical?

**8.** Set up $\iiint_E xyz dV$ where $E$ is bounded by $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 + z^2 = z$ in spherical.

**9.** Set up the integral to find the mass of the solid bounded by $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$ with constant density 2.

**10.** * Set up the integral to the volume of the smaller wedge cut from a sphere of radius 2 by two planes that intersect in a diameter of the sphere at an angle of $\frac{\pi}{6}$.

**11.** * Write $\iiint_E f(x, y, z)dV$ as a triple integral, where $E$ lies above the parabaloid $z = x^2 + y^2$ and below $z = 2y$.

In the following problems, actually evaluate the integrals. For the last, remember improper integration.

**12.**

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{2} xz\,dz\,dx\,dy$$

**13.**

$$\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} xy\,dz\,dy\,dx$$

**14.**

$$\iiint_E \frac{1}{(x^2+y^2+z^2)^2}\,dV$$

where $E = \{(x,y,z) \mid x^2+y^2+z^2 \geq 1\}$.