MATH 21A

Extrema to the Extreme

1. **Warm up:** Consider the function $f(x) = 3x^4 + 5x^3$.

   (a) What are the critical points of the function?

   (b) Classify the critical points.

2. Find and classify the critical points of $h(x, y) = 5 - x^3 + y^3 + 3xy$.

3. Find the three positive numbers that add up to 100 that have the maximum product.

4. Find the minimum distance from $(1, 2, 3)$ to a point on the plane $x - y + z = 4$. *Hint: A little time spent thinking about the function you want to maximize will help here...*

5. Find and classify the critical points of $g(x, y) = \sin x - \sin y$.

6. Find and classify the critical points of $g(x, y) = -(x^2 - 1)^2 - (x^2 y - x - 1)^2$. Is there something strange here? Why? *Hint: The answer to the first question is "Yes."*

7. We want to extremize $f(x, y) = 1 + 4x^2 - 5y^2$ on the triangle with vertices $(0, 0)$, $(2, 0)$, and $(0, 3)$ (boundaries and interior included).

(a) Draw the region and find the equation of the lines on the boundary.

(b) Are there any critical points within the region?

(c) Take the line $x = 0$ (one of the boundaries); are there any critical points of $g(y) = f(0, y)$ in the region described?

(d) Repeat the above with the other two lines.

(e) Take the critical points you have found and the vertices of the triangle and evaluate f at them.

(f) What are the largest and smallest values you found? Congratulations: those are the maximum and minimum values on the triangle! Why?

8. Find the extremal values of $f(x, y) = xy^2 - x^2 - 2y^2$ on the region $\{(x, y) \mid |x| \leq 1 \text{ and } |y| \leq 1\}$.
Hint: draw the region and use the previous problem as a guide!

9. Find and classify the critical points of

$$f(x, y) = x^2y^2 - 2xy^2 - 2x^2y + 4xy - 3y^2 + 6y$$