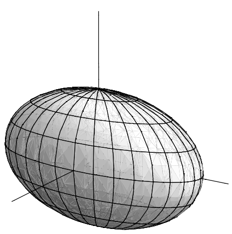
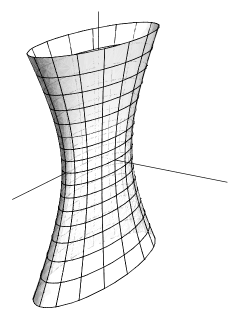
Quadric Surfaces
September 17

Six basic types of quadric surfaces:

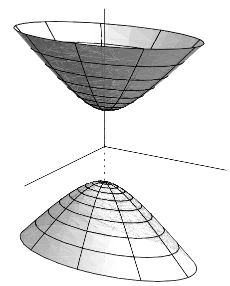
1. ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
2. hyperboloid of one sheet: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
3. hyperboloid of two sheets: $-\frac{x^2}{a^2} + -\frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
4. cone: $\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
5. elliptic paraboloid: $\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
6. hyperbolic paraboloid: $\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$



(1)



(2)



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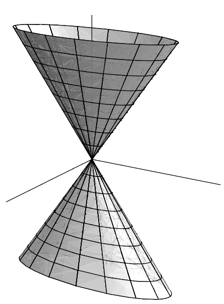

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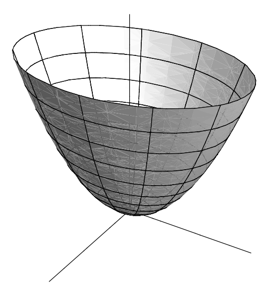
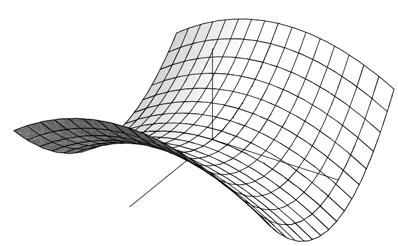


(5)



(6)

When signs, the picture rotates. Typically, the "odd variable out" in terms of sign indicates which axis the shape "opens along."

Quick reminder:

- $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = r$ describes ...
  - an ellipse if $r > 0$.
  - a point if $r = 0$ (we consider this a "degenerate" ellipse).
  - nothing if $r < 0$.
- $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = r$ describes ...
  - a hyperbola if $r \neq 0$.
  - a pair of lines if $r = 0$ (we consider this a "degenerate" hyperbola).
- $y = ax^2 + b$ describes a parabola.
- Similar to completing the square, each of these surfaces can be shifted (and the presence of, say x and x^2 terms indicates this has been done.
  If we know the graph of $f(x, y, z) = 0$, then the graph of $f(x - a, y - b, z - c) = 0$ looks like the same surface shifted a in the x-direction, b in the y-direction, c in the z-direction.

We won't have time to do the following in class, but the exercises below should give you an indication of <u>why</u> the surfaces look like they do.

1. For each surface, describe the traces of the surface in $x = k$, $y = k$, and $z = k$. Then identify what type of quadric it is and sketch it.

   (a) $\dfrac{x^2}{9} - \dfrac{y^2}{16} = z.$

   - Traces in $x = k$:
   - Traces in $y = k$:
   - Traces in $z = k$:

   (b) $\dfrac{x^2}{4} + \dfrac{y^2}{25} + \dfrac{z^2}{9} = 1.$

   - Traces in $x = k$:
   - Traces in $y = k$:
   - Traces in $z = k$:

   (c) $\dfrac{x^2}{4} + \dfrac{y^2}{9} = \dfrac{z}{2}.$

   - Traces in $x = k$:
   - Traces in $y = k$:
   - Traces in $z = k$:

   (d) $\dfrac{z^2}{4} - x^2 - \dfrac{y^2}{4} = 1.$

   - Traces in $x = k$:
   - Traces in $y = k$:
   - Traces in $z = k$:

   (e) $x^2 + \dfrac{y^2}{9} = \dfrac{z^2}{16}.$

   - Traces in $x = k$:
   - Traces in $y = k$:

- Traces in $z = k$:

(f) $\dfrac{x^2}{9} + y^2 - \dfrac{z^2}{16} = 1.$

- Traces in $x = k$:
- Traces in $y = k$:
- Traces in $z = k$:

2. Sketch the surface $9y^2 + 4z^2 = 36$. What type of surface is it?

3. Sketch the surface $y^2 + 2y + z^2 = x^2$. What type of quadric surface is it?

4. What type of quadric surface is $4x^2 - y^2 + z^2 + 9 = 0$?