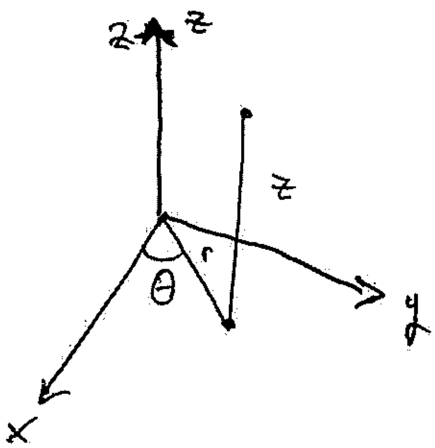
Cylindrical

Polar in plane, height z

$$(r, \theta, z) \longmapsto (r\cos\theta, r\sin\theta, z)$$

$$(x, y, z)$$

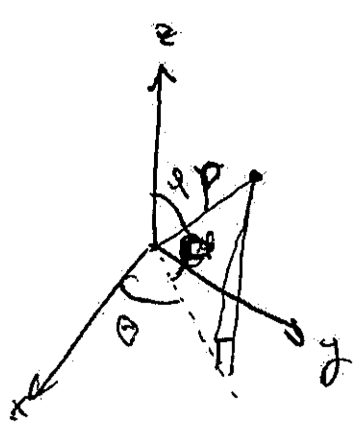


Spherical

True generalization of polar

$$(\rho, \theta, \varphi) \longmapsto \cancel{(\rho\cos\theta\sin\varphi, \rho\sin\theta\sin\varphi, \rho\cos\varphi)}$$

$$\cancel{\left(\varphi \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]\right)} \quad (\varphi \in [0, \pi])$$



$$\longrightarrow (\rho\cos\theta\sin\varphi, \rho\sin\theta\sin\varphi, \rho\cos\varphi)$$

$$\theta = \begin{cases} \cos\theta = \dfrac{x}{\rho\sin\varphi} = \dfrac{-12}{(8\sqrt{6})(\frac{\sqrt{3}}{2})} = \dfrac{-3}{3\sqrt{2}} = \dfrac{-1}{\sqrt{2}} \\ \sin\theta = \dfrac{y}{\rho\sin\varphi} = \dfrac{12}{(8\sqrt{6})(\frac{\sqrt{3}}{2})} = \dfrac{1}{\sqrt{2}} \end{cases}$$

$$\theta = \frac{3\pi}{4}$$

$$\left(8\sqrt{6}, \frac{3\pi}{4}, \frac{2\pi}{3}\right)$$

A <u>parameterization</u> of a surface is

$\vec{r}(u,v)$ s.t. the surface is

$$\{(x,y,z) \in \mathbb{R}^3 \mid \langle x,y,z \rangle = \vec{r}(u,v)\}$$

Seen a bunch before

- <u>Planes</u> Given a plane $3x+2y+z=0$, it is parameterized by

$$\vec{r}(u,v) = \langle u, v, -2v - 3u \rangle$$

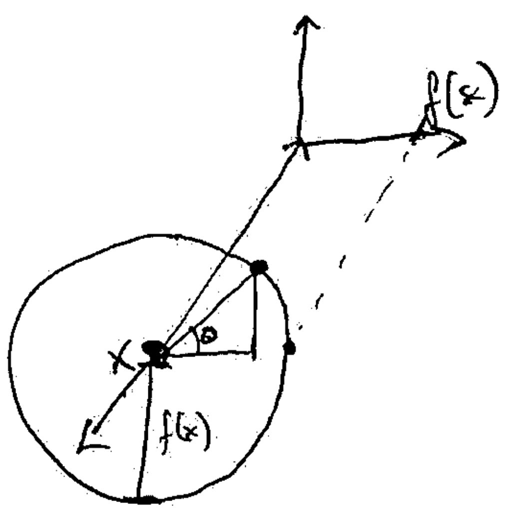
- <u>Graphs</u> Generally, graphs have a very easy parameterization:

$$\vec{r}(u,v) = \langle u, v, f(u,v) \rangle$$

- Surfaces of revolution

Have our ~~f~~favorite $y = f(x)$ in xy-plane.

Want to rotate about <u>x</u>-axis



$$x = x$$
$$y = f(x)\cos\theta$$
$$z = f(x)\sin\theta$$

parameterized by
$$\vec{r}(x,\theta) = \langle x, f(x)\cos\theta, f(x)\sin\theta \rangle$$

Change based on axis