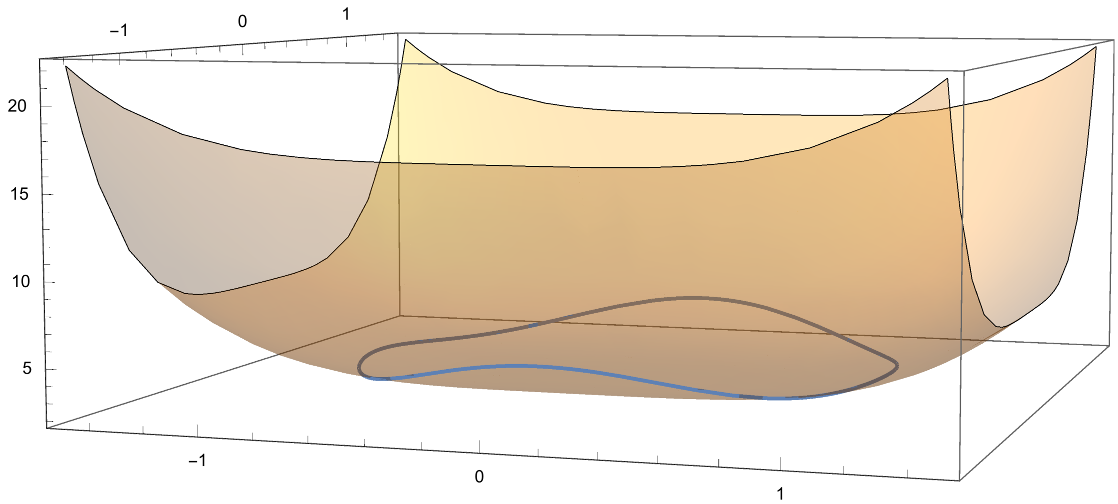
In class, we found the maximum and minimum values of $f(x, y) = x^4 + 3y^4 + 2$ given $x^2 + y^2 = 1$. This corresponds to finding the highest and lowest point of the blue curve in the following picture.

In[30]:=
```
Show[
 Plot3D[x^4 + 3 y^4 + 2, {x, -1.5, 1.5},
  {y, -1.5, 1.5}, Mesh → None, PlotStyle → Opacity[0.3]],
 ParametricPlot3D[{{Cos[t], Sin[t], Cos[t]^4 + 3 Sin[t]^4 + 2},
   {Cos[t], Sin[t], 0}}, {t, 0, 2 Pi}]
 , ImageSize → Large]
```

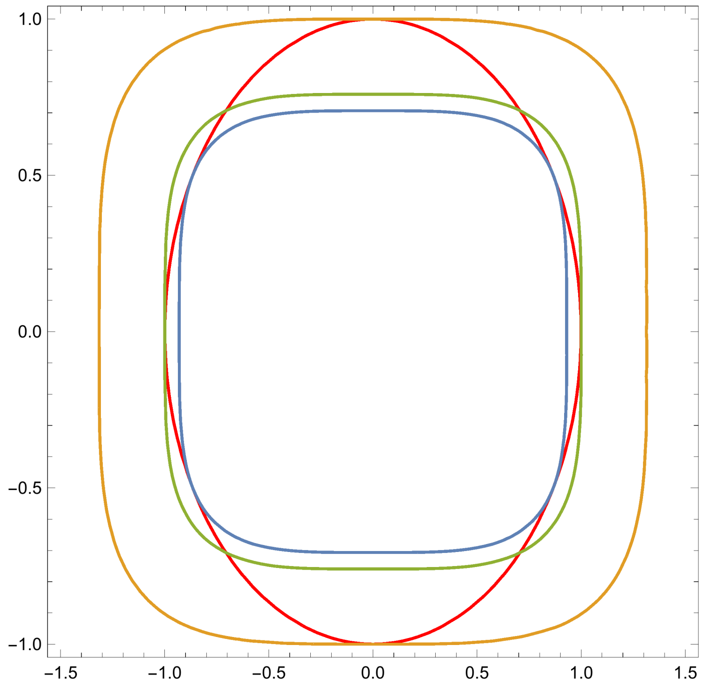
Out[30]=



Using Lagrange multipliers, we found that the test points gave us values 5, 3, and $\frac{11}{4}$. The level curves of f at these values along with the constraint curve are graphed below.

In[16]:=
```
Show[ContourPlot[x^2 + y^2 == 1, {x, -1.5, 1.5},
  {y, -1, 1}, ColorFunction → Function[{x, y}, Red]],
 ContourPlot[{x^4 + 3 y^4 + 2 == 11/4, x^4 + 3 y^4 + 2 == 5, x^4 + 3 y^4 + 2 == 3},
  {x, -2, 2}, {y, -2, 2}]]
```

Out[16]=



We see that each of the level curves of of f has points of tangency with the As the value k in the level curve f(x,y)=k decreases, the rounded circles representing these level curves also shrink. Each of the graphed level curves have a point of tangency with the red ellipse.

If the “radius” of the yellow level curve f(x,y)=5 were increased a little, it would no longer intersect the level curve $x^2 + y^2 = 1$; this agrees with the math that showed 5 was the maximum value of f on this curve.

If the “radius” of the blue level curve $f(x, y) = \frac{11}{4}$ were decreased a little, it would no longer intersect the level curve $x^2 + y^2 = 1$; this agrees with the math that showed $\frac{11}{4}$ was the minimum value of f on this curve.

If the “radius” of the green level curve f(x,y)=3 were changed, it would still intersect the level curve $x^2 + y^2 = 1$; this agrees with the conclusion that 3 was neither the maximum or the minimum value.