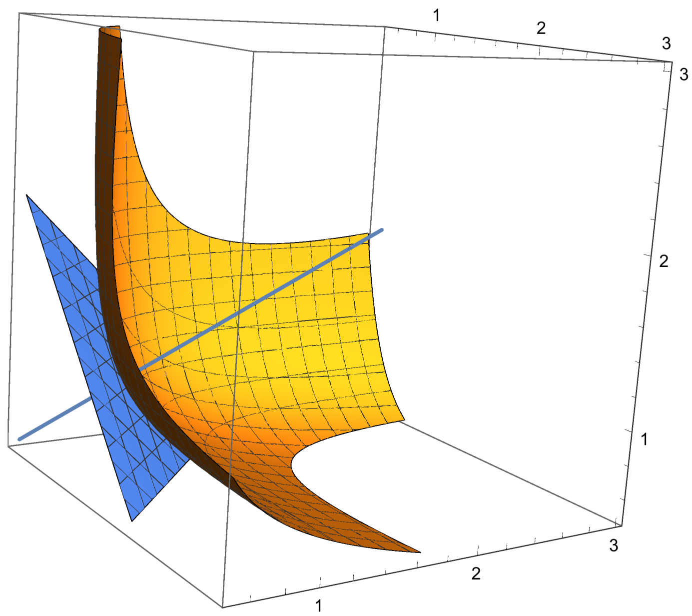
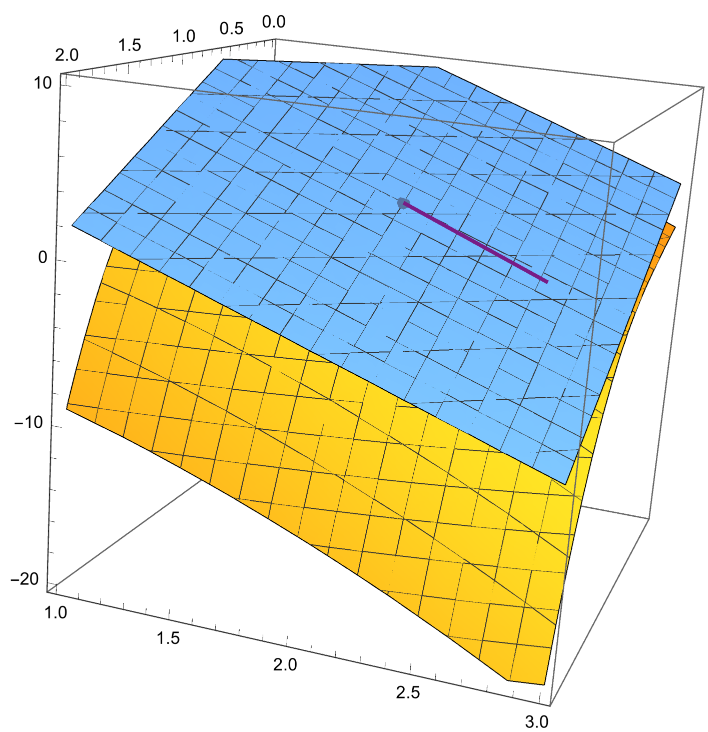
## We compare the meaning of the gradient in the first and last

```
Show[ContourPlot3D[{x^y y^z z^x == 1, x + y + z == 3}, {x, .5, 3}, {y, .5, 3},
  {z, .5, 3}], ParametricPlot3D[{1 + t, 1 + t, 1 + t}, {t, -.5, 1}], ListPointPlot3D[
  {{1, 1, 1}}, Filling -> Bottom, PlotStyle → PointSize[Large], PlotStyle → Black]]
```



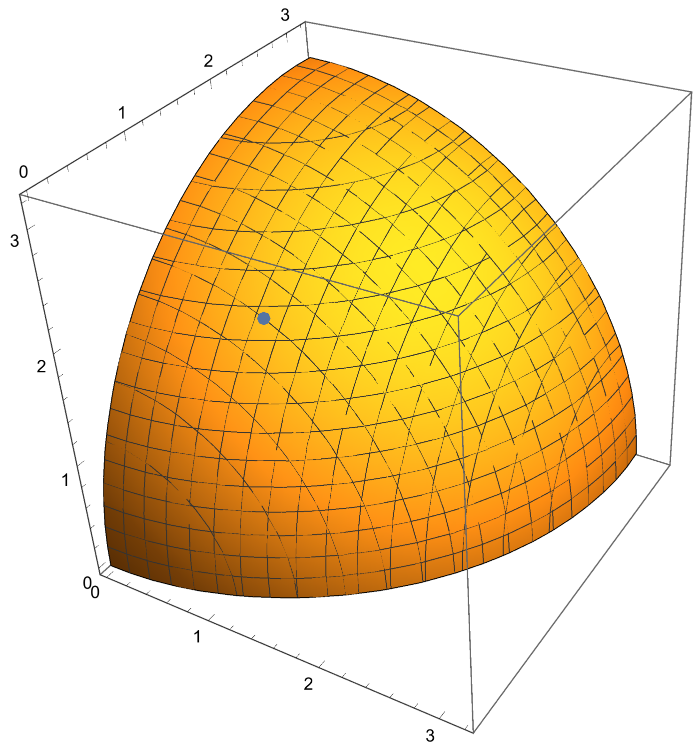

```
Show[ContourPlot3D[{z == 10 - x^4 - x y - y^2, -6 (x - 1) + -5 (y - 2) + - (z - 3) == 0},
  {x, 0, 2}, {y, 1, 3}, {z, -20, 10}], ListPointPlot3D[{{1, 2, 3}},
  Filling -> Bottom, PlotStyle → PointSize[Large], PlotStyle → Black],
 ParametricPlot3D[{{1 - 6 t, 2 - 5 t, 3}}, {t, -2, 2}, ColorFunction → "Rainbow"]]
```



# What is the meaning of the gradient in the second graph?

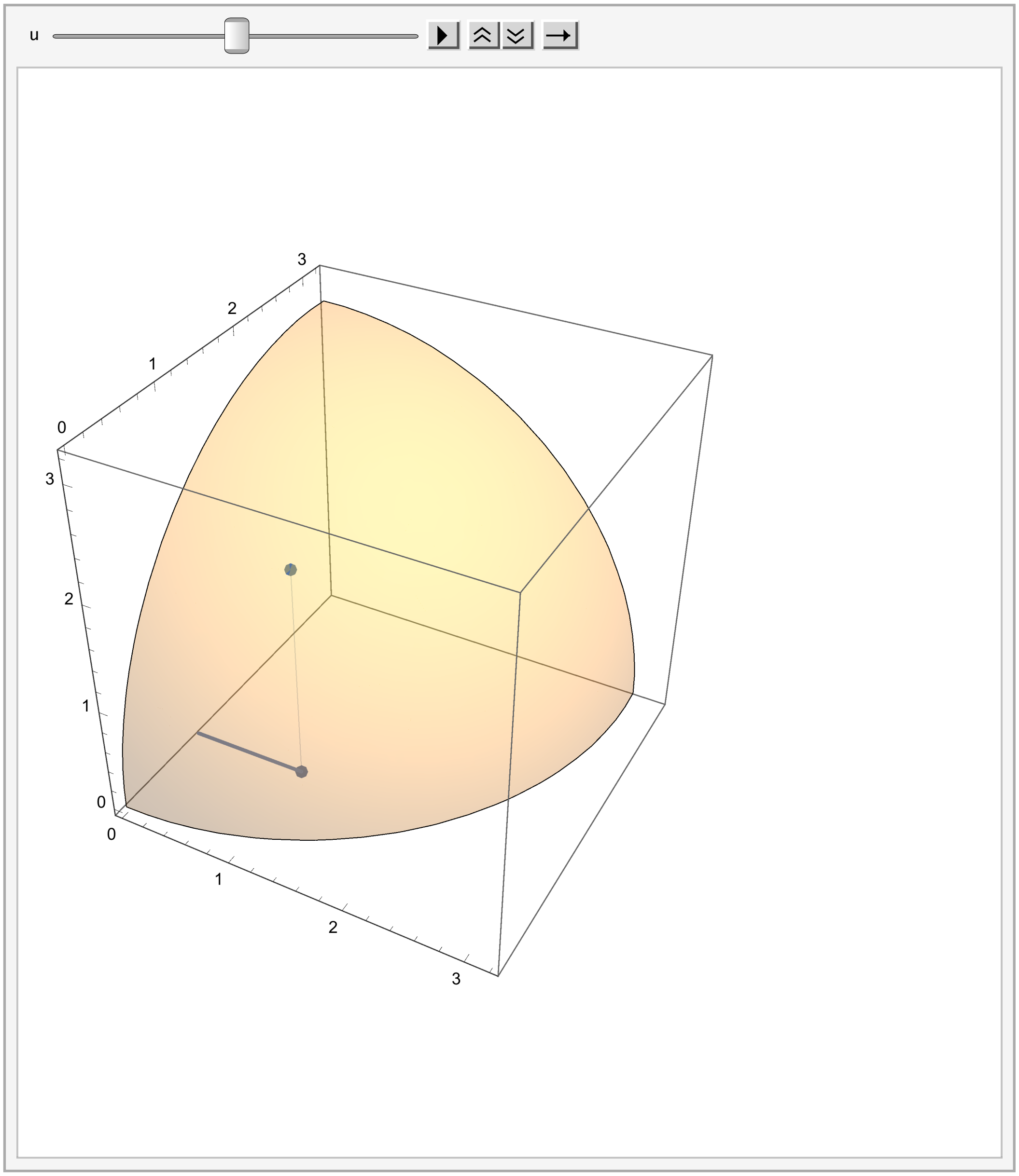
**Really want sphere at origin, but we rotate it so the view is better.**

```
Show[ContourPlot3D[(y - 3)^2 + x^2 + z^2 == 3^2, {x, 0, 3.2}, {y, 0, 3.2}, {z, 0, 3.2}],
 ListPointPlot3D[{{1, 1, 2}}, Filling -> Bottom,
  PlotStyle → PointSize[Large], PlotStyle → Black]]
```



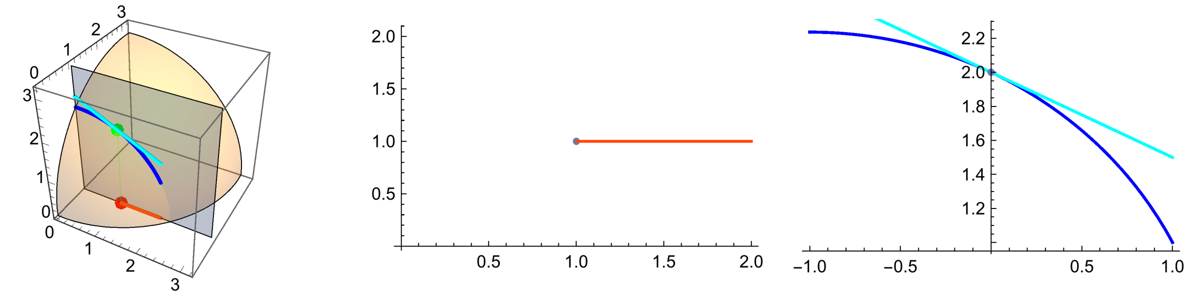

```
Clear[u]
```

```
Animate[Show[ContourPlot3D[(y - 3)^2 + x^2 + z^2 == 3^2, {x, 0, 3.2},
    {y, 0, 3.2}, {z, 0, 3.2}, Mesh → None, ContourStyle → Opacity[0.3]],
  ListPointPlot3D[{{1, 1, 2}, {1, 1, 0}}, Filling -> Bottom,
   PlotStyle → PointSize[Large], PlotStyle → Black],
  ParametricPlot3D[{1 + Cos[u] t, 1 + Sin[u] t, 0}, {t, 0, 1}]], {u, 0, 2 Pi}]
```



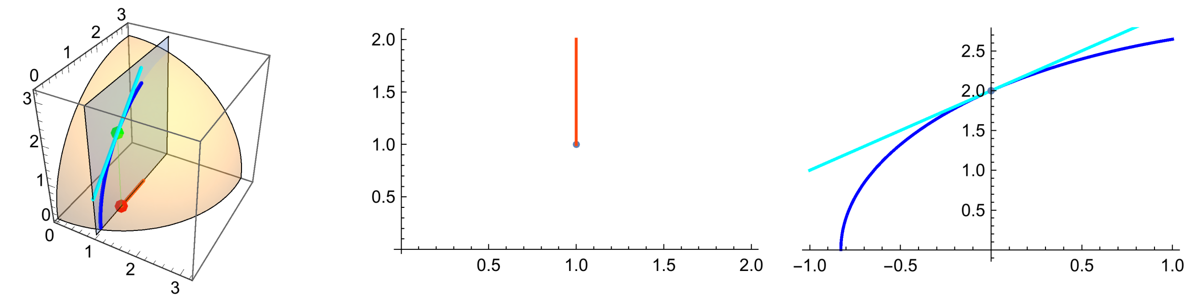

The partial derivatives are directional derivatives in different directions. The pa

**f[0]**



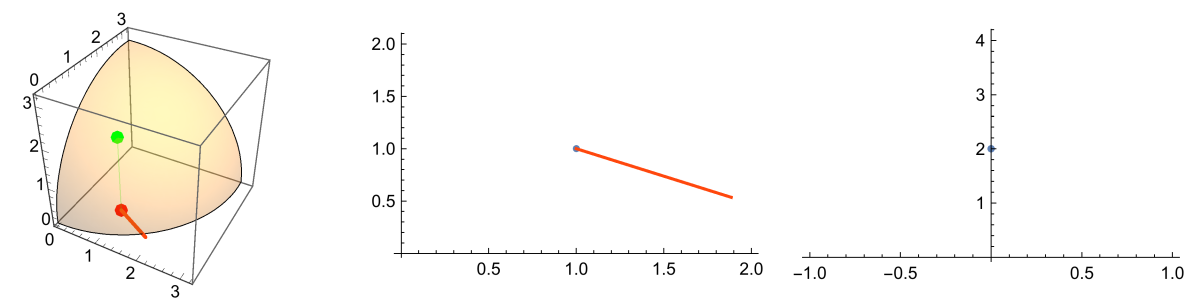
**The partial with respect to x computes the slope of the green line when the red vect**

**f[Pi / 2]**



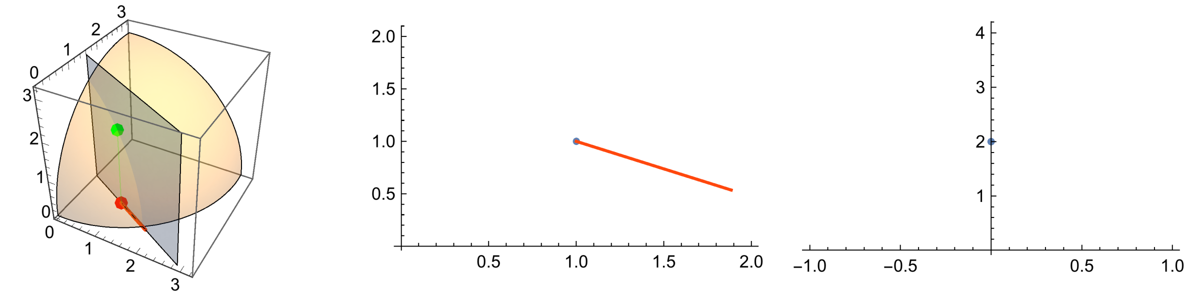
**But these are just two directions.  We can define the directional derivative at a po**

**f1[5.8]**



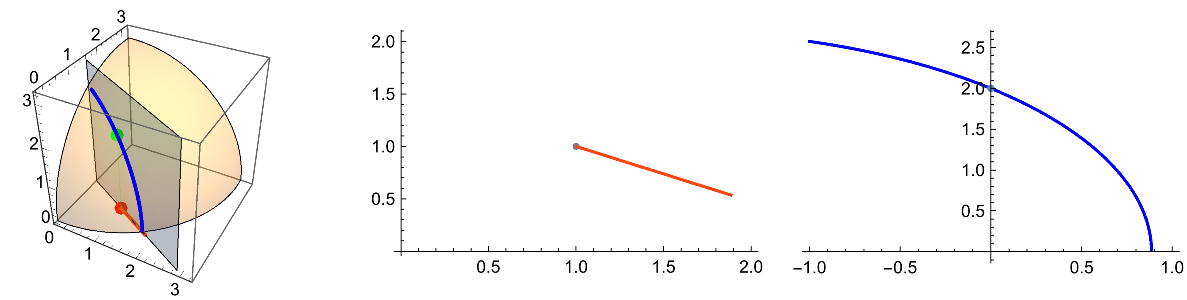
**We add in the plane that contains the point, sticks straight up, and contains the di**

**f2[5.8]**



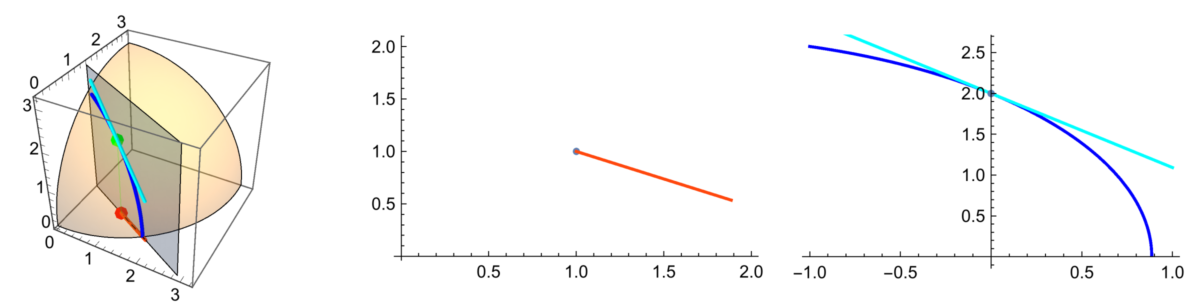

**The intersection of the plane and the graph defines a curve in the plane.  The grap**

**f3[5.8]**



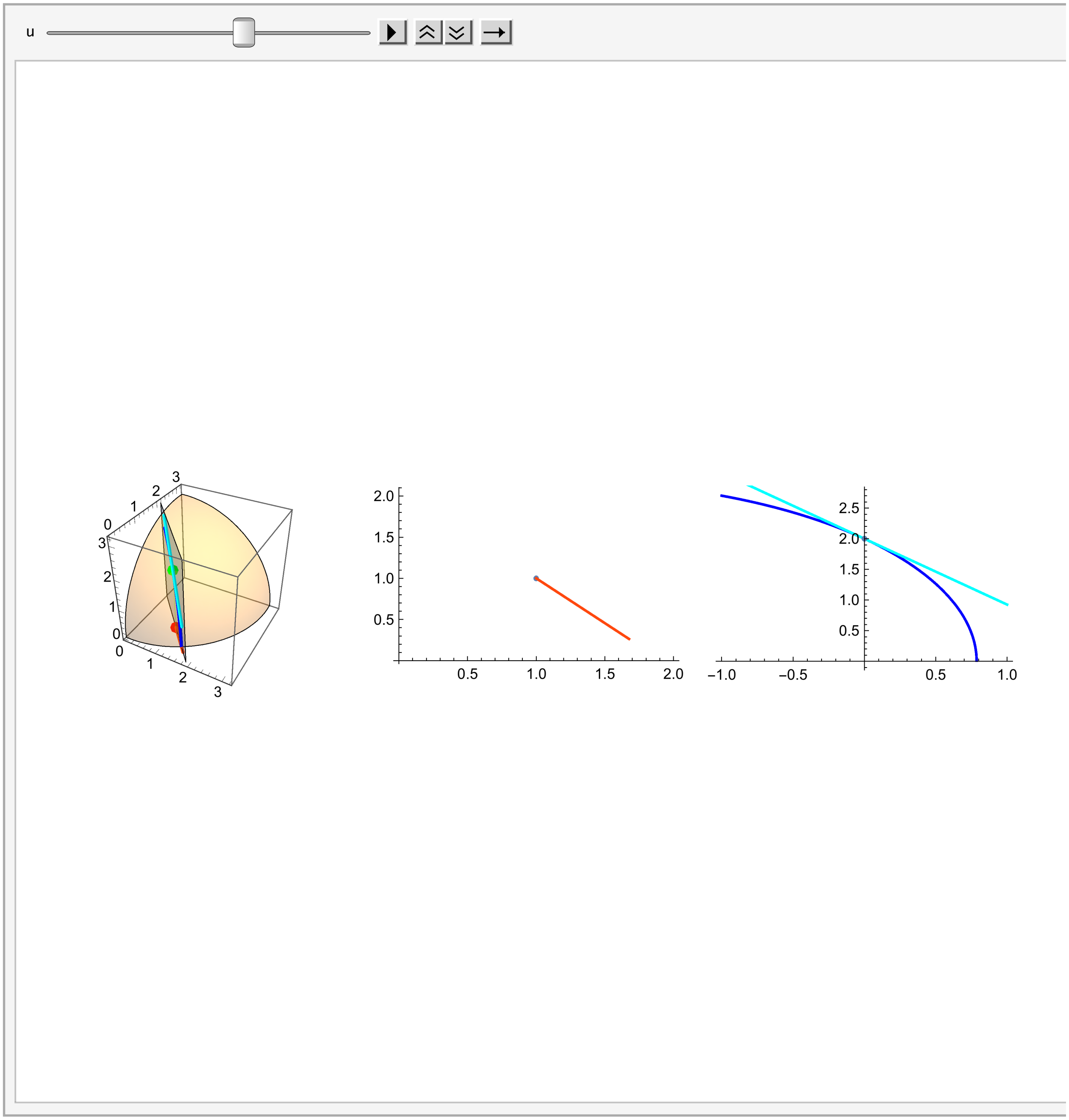

**The directional derivative computes the derivative of the graph on the right.  More**

**f[5.8]**




**Now we can animate this**

```
Animate[f[u], {u, 0, 6 Pi}]
```



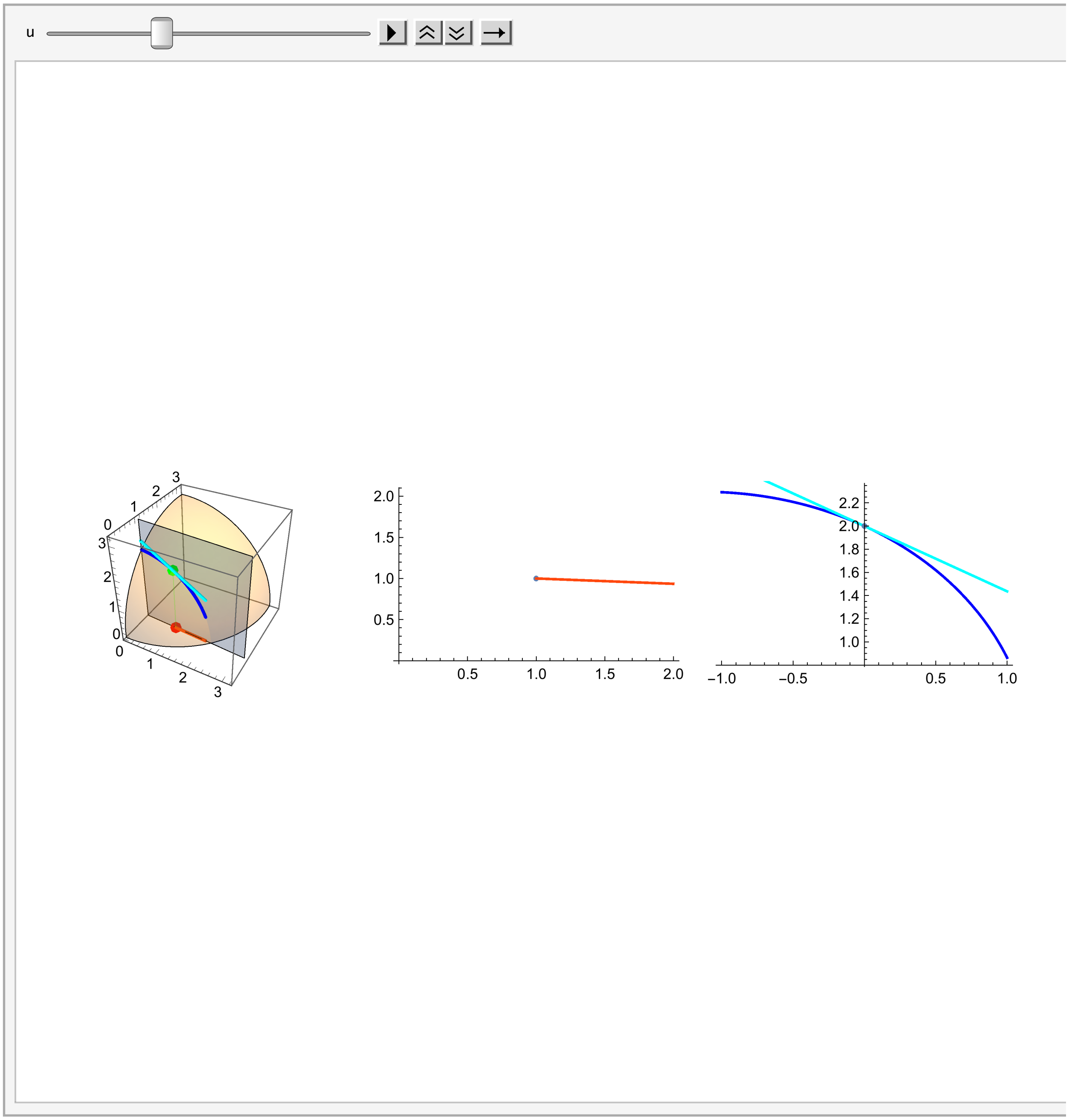
```
In[32]:= f[u_] := GraphicsRow[{
    Show[ContourPlot3D[{(y - 3)^2 + x^2 + z^2 == 3^2, -Sin[u] (x - 1) + Cos[u] (y - 1) == 0}, {x, 0,
       3.2}, {y, 0, 3.2}, {z, 0, 3.2}, Mesh → None, ContourStyle → Opacity[0.3]],
     ListPointPlot3D[{{1, 1, 2}, {1, 1, 0}}, Filling -> Bottom,
      PlotStyle → PointSize[Large], ColorFunction → Function[{x, y, z}, Hue[z / 3]]],
     ParametricPlot3D[{1 + Cos[u] t, 1 + Sin[u] t, 0}, {t, 0, 1},
      ColorFunction → Function[{x, y, z}, Hue[0.04, 0.95, 1]]], ParametricPlot3D[
      {1 + Cos[u] s, 1 + Sin[u] s, (9 - (1 + Cos[u] s)^2 - (1 + Sin[u] s - 3)^2)^(1/2)},
      {s, -1, 1}, ColorFunction → Function[{x, y, z}, Hue[2 / 3]]],
     ParametricPlot3D[{1 + Cos[u] s, 1 + Sin[u] s, (-Cos[u]/2 + Sin[u]) s + 2},
      {s, -1, 1}, ColorFunction → Function[{x, y, z}, Hue[1 / 2]]]],
    Show[ListPlot[{{1, 1}}, ColorFunction → Red],
     ParametricPlot[{1 + Cos[u] t, 1 + Sin[u] t}, {t, 0, 1},
      ColorFunction → Function[{x, y}, Hue[0.04, 0.95, 1]]]],
    Show[Plot[(9 - (1 + Cos[u] a)^2 - (1 + Sin[u] a - 3)^2)^(1/2), {a, -1, 1},
      ColorFunction → Function[{x, y}, Hue[2 / 3]]], Plot[(-1/2 Cos[u] + Sin[u]) b + 2,
      {b, -1, 1}, ColorFunction → Function[{x, y}, Hue[1 / 2]]], ListPlot[{{0, 2}}]]
   }, ImageSize → Full]

f1[u_] := GraphicsRow[{
    Show[ContourPlot3D[{(y - 3)^2 + x^2 + z^2 == 3^2}, {x, 0, 3.2},
      {y, 0, 3.2}, {z, 0, 3.2}, Mesh → None, ContourStyle → Opacity[0.3]],
     ListPointPlot3D[{{1, 1, 2}, {1, 1, 0}}, Filling -> Bottom,
      PlotStyle → PointSize[Large], ColorFunction → Function[{x, y, z}, Hue[z / 3]]],
     ParametricPlot3D[{1 + Cos[u] t, 1 + Sin[u] t, 0}, {t, 0, 1},
      ColorFunction → Function[{x, y, z}, Hue[0.04, 0.95, 1]]] ],
    Show[ListPlot[{{1, 1}}, ColorFunction → Red],
     ParametricPlot[{1 + Cos[u] t, 1 + Sin[u] t}, {t, 0, 1},
      ColorFunction → Function[{x, y}, Hue[0.04, 0.95, 1]]]],
    Show[ListPlot[{{0, 2}}]]
   }, ImageSize → Full]
```

```
f2[u_] := GraphicsRow[{
   Show[ContourPlot3D[{(y - 3)^2 + x^2 + z^2 == 3^2, -Sin[u] (x - 1) + Cos[u] (y - 1) == 0}, {x, 0,
      3.2}, {y, 0, 3.2}, {z, 0, 3.2}, Mesh → None, ContourStyle → Opacity[0.3]],
    ListPointPlot3D[{{1, 1, 2}, {1, 1, 0}}, Filling -> Bottom,
     PlotStyle → PointSize[Large], ColorFunction → Function[{x, y, z}, Hue[z / 3]]],
    ParametricPlot3D[{1 + Cos[u] t, 1 + Sin[u] t, 0}, {t, 0, 1},
     ColorFunction → Function[{x, y, z}, Hue[0.04, 0.95, 1]]] ],
   Show[ListPlot[{{1, 1}}, ColorFunction → Red],
    ParametricPlot[{1 + Cos[u] t, 1 + Sin[u] t}, {t, 0, 1},
     ColorFunction → Function[{x, y}, Hue[0.04, 0.95, 1]]]],
   Show[ListPlot[{{0, 2}}]]
  }, ImageSize → Full]
```

```
f3[u_] := GraphicsRow[{
   Show[ContourPlot3D[{(y - 3)^2 + x^2 + z^2 == 3^2, -Sin[u] (x - 1) + Cos[u] (y - 1) == 0},
     {x, 0, 3.2}, {y, 0, 3.2}, {z, 0, 3.2}, Mesh → None,
     ContourStyle → Opacity[0.3]], ListPointPlot3D[{{1, 1, 2}, {1, 1, 0}},
     Filling -> Bottom, PlotStyle → PointSize[Large],
     ColorFunction → Function[{x, y, z}, Hue[z / 3]]], ParametricPlot3D[
     {1 + Cos[u] s, 1 + Sin[u] s, (9 - (1 + Cos[u] s)^2 - (1 + Sin[u] s - 3)^2)^(1/2)},
     {s, -1, 1}, ColorFunction → Function[{x, y, z}, Hue[2 / 3]]],
    ParametricPlot3D[{1 + Cos[u] t, 1 + Sin[u] t, 0}, {t, 0, 1},
     ColorFunction → Function[{x, y, z}, Hue[0.04, 0.95, 1]]] ],
   Show[ListPlot[{{1, 1}}, ColorFunction → Red],
    ParametricPlot[{1 + Cos[u] t, 1 + Sin[u] t}, {t, 0, 1},
     ColorFunction → Function[{x, y}, Hue[0.04, 0.95, 1]]]],
   Show[Plot[(9 - (1 + Cos[u] a)^2 - (1 + Sin[u] a - 3)^2)^(1/2), {a, -1, 1},
     ColorFunction → Function[{x, y}, Hue[2 / 3]]], ListPlot[{{0, 2}}]]
  }, ImageSize → Full]
```

```
Animate[GraphicsRow[{
   Show[ContourPlot3D[{(y - 3)^2 + x^2 + z^2 == 3^2, -Sin[u] (x - 1) + Cos[u] (y - 1) == 0}, {x, 0,
      3.2}, {y, 0, 3.2}, {z, 0, 3.2}, Mesh → None, ContourStyle → Opacity[0.3]],
    ListPointPlot3D[{{1, 1, 2}, {1, 1, 0}}, Filling -> Bottom,
     PlotStyle → PointSize[Large], ColorFunction → Function[{x, y, z}, Hue[z / 3]]],
    ParametricPlot3D[{1 + Cos[u] t, 1 + Sin[u] t, 0}, {t, 0, 1},
     ColorFunction → Function[{x, y, z}, Hue[0.04, 0.95, 1]]], ParametricPlot3D[
     {1 + Cos[u] s, 1 + Sin[u] s, (9 - (1 + Cos[u] s)^2 - (1 + Sin[u] s - 3)^2)^(1/2)},
     {s, -1, 1}, ColorFunction → Function[{x, y, z}, Hue[2 / 3]]],
    ParametricPlot3D[{1 + Cos[u] s, 1 + Sin[u] s, (-Cos[u]/2 + Sin[u]) s + 2},
     {s, -1, 1}, ColorFunction → Function[{x, y, z}, Hue[1 / 2]]]],
   Show[ListPlot[{{1, 1}}, ColorFunction → Red],
    ParametricPlot[{1 + Cos[u] t, 1 + Sin[u] t}, {t, 0, 1},
     ColorFunction → Function[{x, y}, Hue[0.04, 0.95, 1]]]],
   Show[Plot[(9 - (1 + Cos[u] a)^2 - (1 + Sin[u] a - 3)^2)^(1/2), {a, -1, 1},
     ColorFunction → Function[{x, y}, Hue[2 / 3]]], Plot[(-1/2 Cos[u] + Sin[u]) b + 2,
     {b, -1, 1}, ColorFunction → Function[{x, y}, Hue[1 / 2]]], ListPlot[{{0, 2}}]]
  }, ImageSize → Full], {u, 0, 6 Pi}]
```



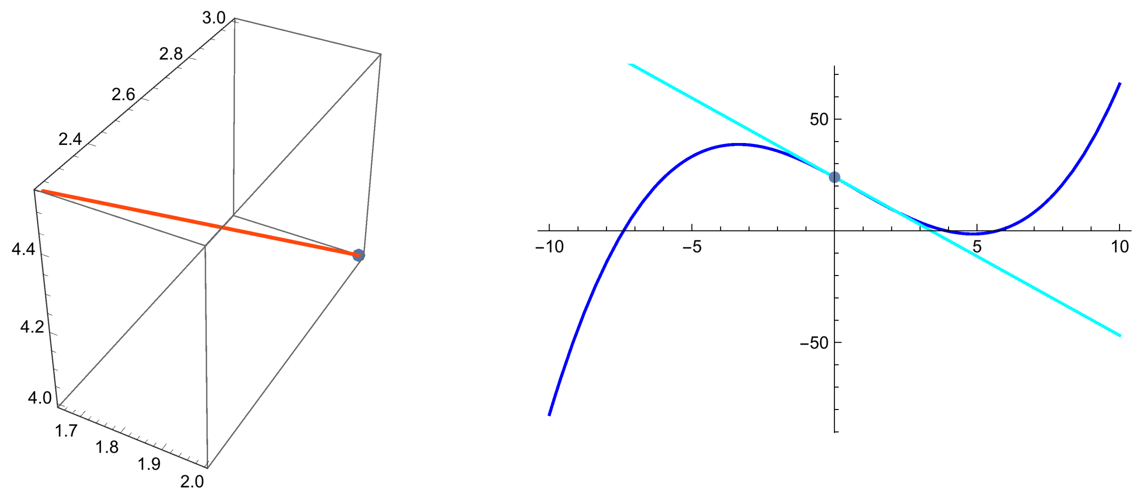

**We want to examine the directional derivative of f (x, y, z) = x y z at (2, 3, 4, 2**

**t = - 2**
**p = 1**

-2

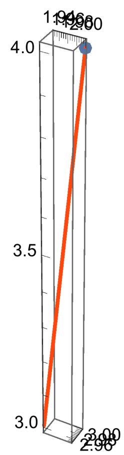
1

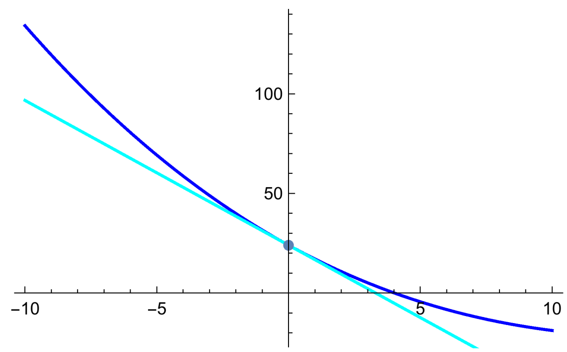
```
GraphicsRow[{
  Show[ParametricPlot3D[{2 + s Cos[t] Sin[p], 3 + s Sin[t] Sin[p], 4 + s Cos[p] },
    {s, 0, 1}, ColorFunction → Function[{x, y}, Hue[0.04, 0.95, 1]]],
   ListPointPlot3D[{{2, 3, 4}}, PlotStyle → PointSize[Large]]],
  Show[Plot[(2 + r Cos[t] Sin[p]) * (3 + r Sin[t] Sin[p]) * (4 + r Cos[p]),
    {r, -10, 10}, ColorFunction → Function[{x, y}, Hue[2 / 3]]],
   Plot[(12 (Cos[t] Sin[p]) + 8 (Sin[t] Sin[p]) + 6 (Cos[p])) r + 24, {r, -10, 10},
    ColorFunction → Function[{x, y}, Hue[1 / 2]]], ListPlot[{{0, 24}}]]
 }, ImageSize →
  Full]
```



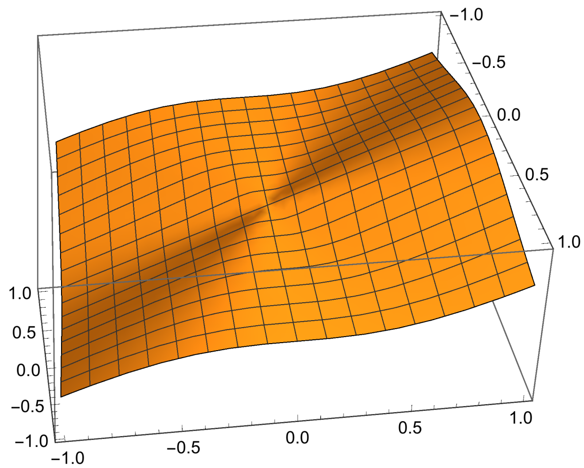

```
In[31]:= g[t_, p_] := GraphicsRow[{
    Show[ParametricPlot3D[{2 + s Cos[t] Sin[p], 3 + s Sin[t] Sin[p], 4 + s Cos[p] },
      {s, 0, 1}, ColorFunction → Function[{x, y}, Hue[0.04, 0.95, 1]]],
     ListPointPlot3D[{{2, 3, 4}}, PlotStyle → PointSize[Large]]],
    Show[Plot[(2 + r Cos[t] Sin[p]) * (3 + r Sin[t] Sin[p]) * (4 + r Cos[p]),
      {r, -10, 10}, ColorFunction → Function[{x, y}, Hue[2 / 3]]],
     Plot[(12 (Cos[t] Sin[p]) + 8 (Sin[t] Sin[p]) + 6 (Cos[p])) r + 24, {r, -10, 10},
      ColorFunction → Function[{x, y}, Hue[1 / 2]]], ListPlot[{{0, 24}}]]
   }, ImageSize → Full]

Animate[g[u, 4 Cos[u]], {u, 0, 10}]
```




$$\text{Plot3D}\left[\frac{y^3}{x^2+y^2}, \{x, -1, 1\}, \{y, -1, 1\}\right]$$



−1.0
−0.5
0.0
0.5
1.0
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0