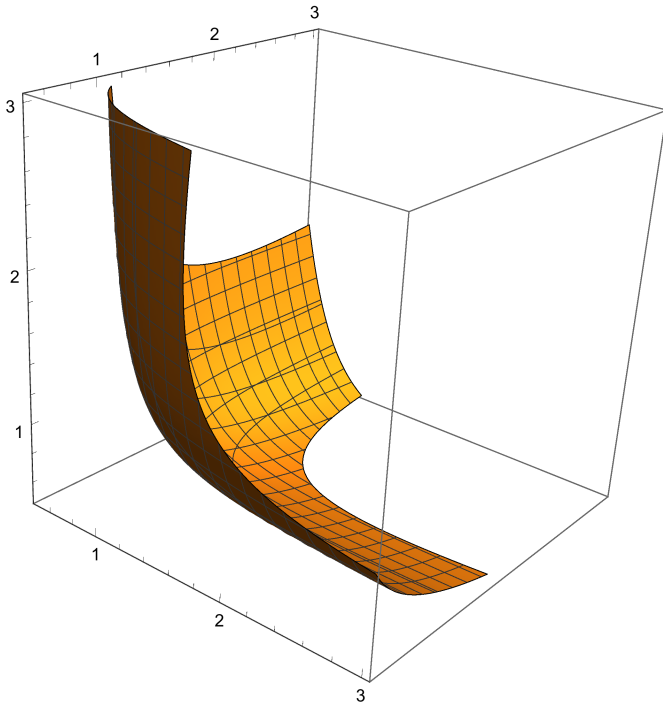
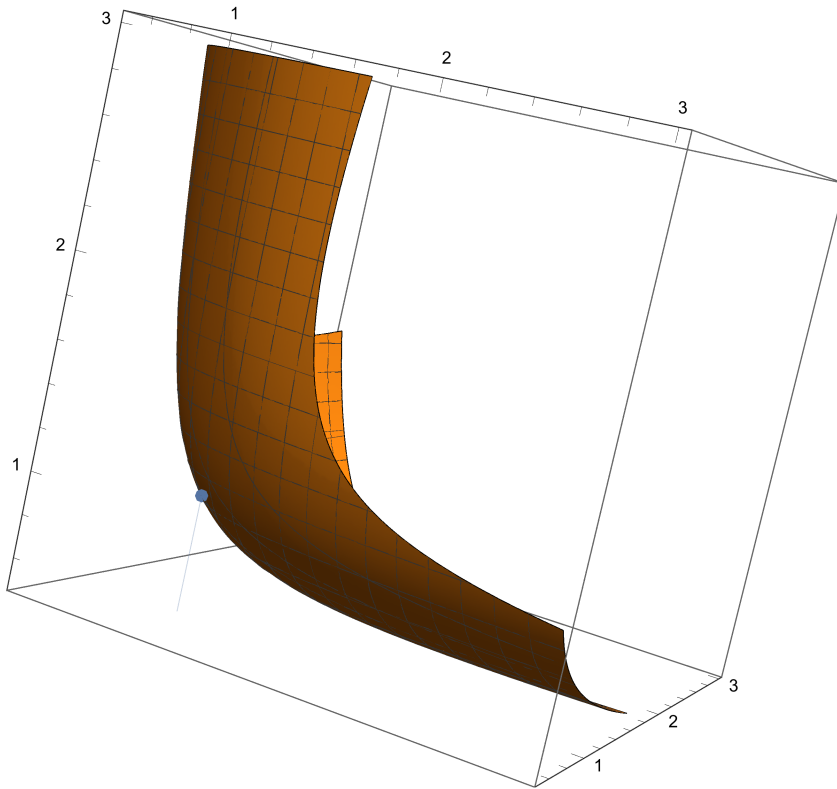


We find the tangent plane to $x^y y^z z^x = 1$ at $(1,1,1)$ by computing the g

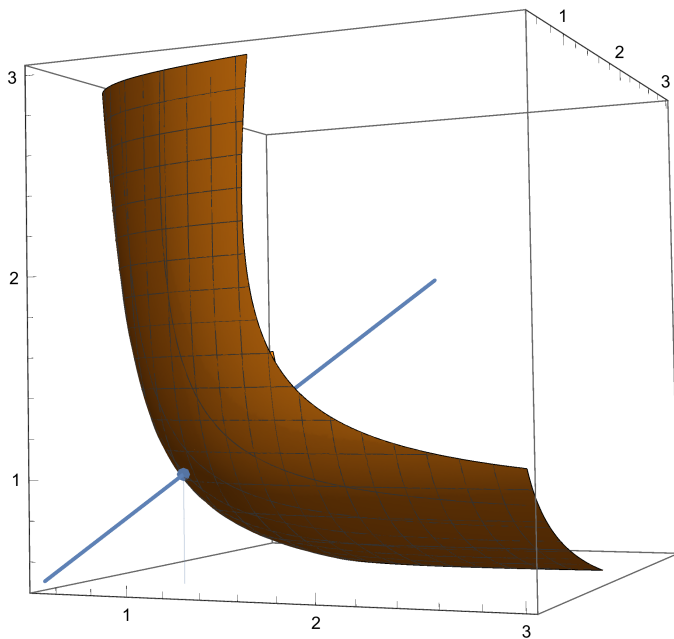
```
ContourPlot3D[x^y y^z z^x == 1, {x, .5, 3}, {y, .5, 3}, {z, .5, 3}]
```



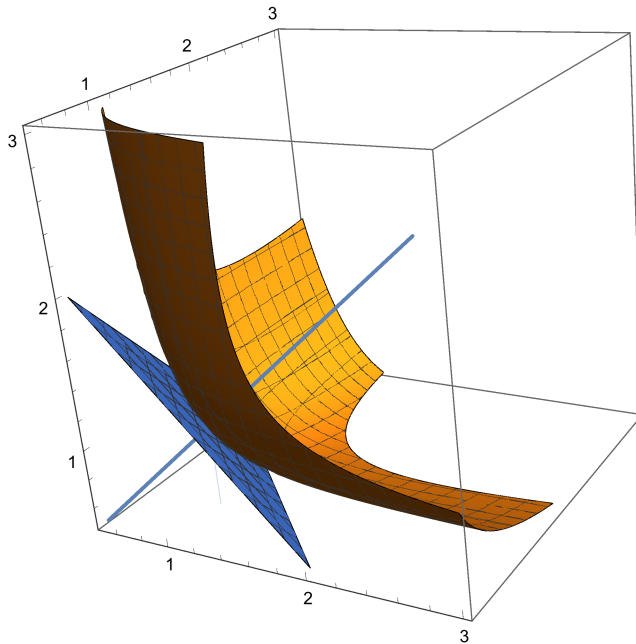
```
Show[ContourPlot3D[x^y y^z z^x == 1, {x, .5, 3}, {y, .5, 3}, {z, .5, 3}],
ListPointPlot3D[{{1, 1, 1}}, Filling -> Bottom,
PlotStyle -> PointSize[Large], PlotStyle -> Black]]
```



```
Show[ContourPlot3D[x^y y^z z^x == 1, {x, .5, 3}, {y, .5, 3}, {z, .5, 3}],
ParametricPlot3D[{1+t, 1+t, 1+t}, {t, -.5, 1}], ListPointPlot3D[{{1, 1, 1}},
Filling -> Bottom, PlotStyle -> PointSize[Large], PlotStyle -> Black]]
```

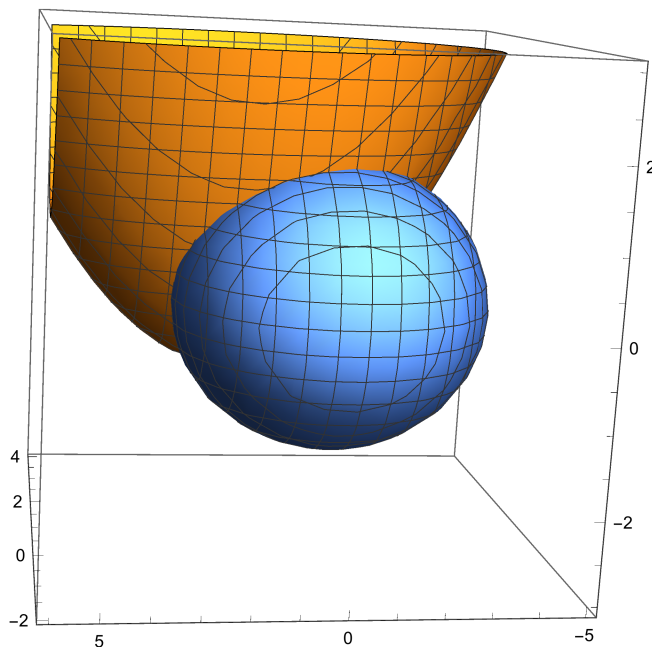


```
Show[ContourPlot3D[{x^y y^z z^x == 1, x + y + z == 3}, {x, .5, 3}, {y, .5, 3},
{z, .5, 3}], ParametricPlot3D[{1 + t, 1 + t, 1 + t}, {t, -.5, 1}], ListPointPlot3D[
{{1, 1, 1}}, Filling -> Bottom, PlotStyle -> PointSize[Large], PlotStyle -> Black]]
```

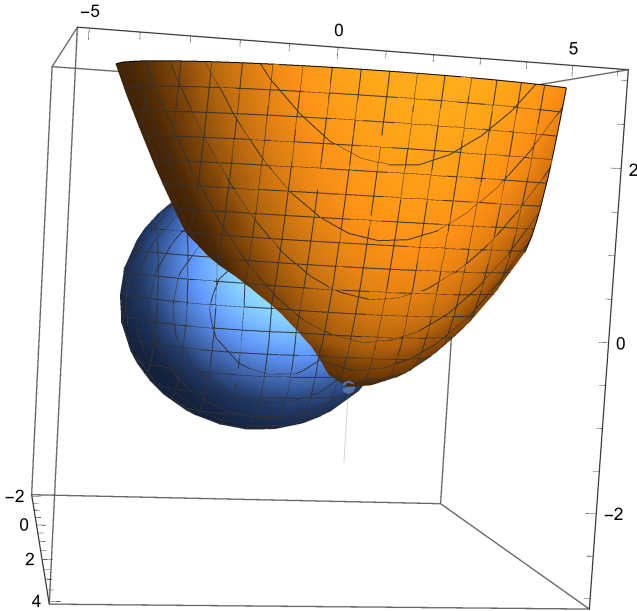


We check if two surfaces are perpendicular by checking their

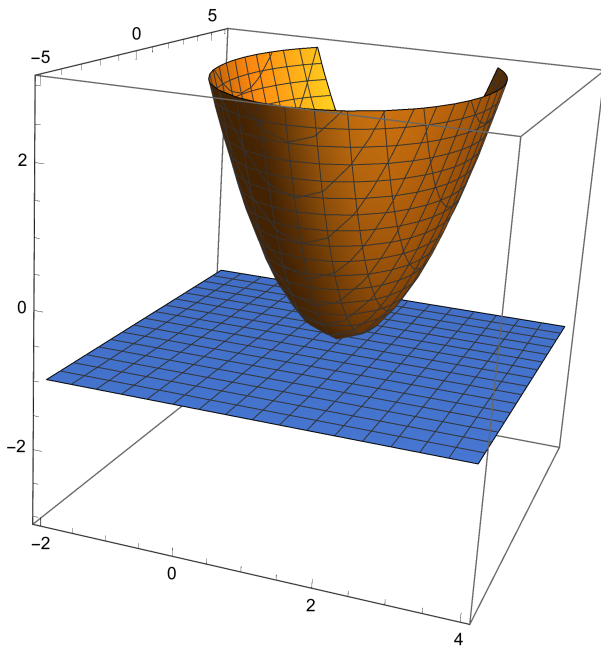
```
ContourPlot3D[{(x - 1)^2 + ((y - 2)/3)^2 == (z + 1), x^2 + ((y/2)^2 + z^2 == 3},
{x, -2, 4}, {y, -5, 6}, {z, -3, 3}]
```



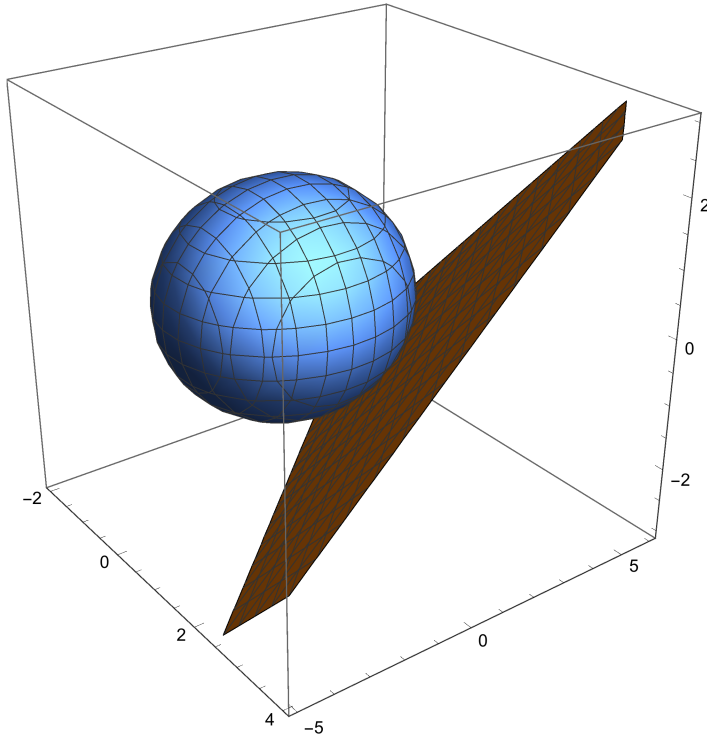
```
Show[ContourPlot3D[{(x - 1)^2 + ((y - 2)/3)^2 == (z + 1), x^2 + (y/2)^2 + z^2 == 3},
{x, -2, 4}, {y, -5, 6}, {z, -3, 3}], ListPointPlot3D[{{1, 2, -1}},
Filling -> Bottom, PlotStyle -> PointSize[Large], PlotStyle -> Black]
```



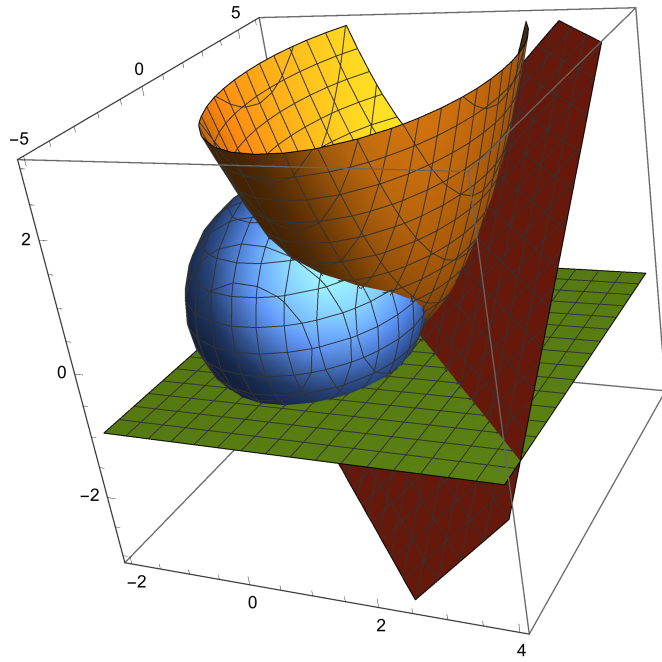
```
ContourPlot3D[{(x - 1)^2 + ((y - 2)/3)^2 == (z + 1), -z == 1},
{x, -2, 4}, {y, -5, 6}, {z, -3, 3}]
```



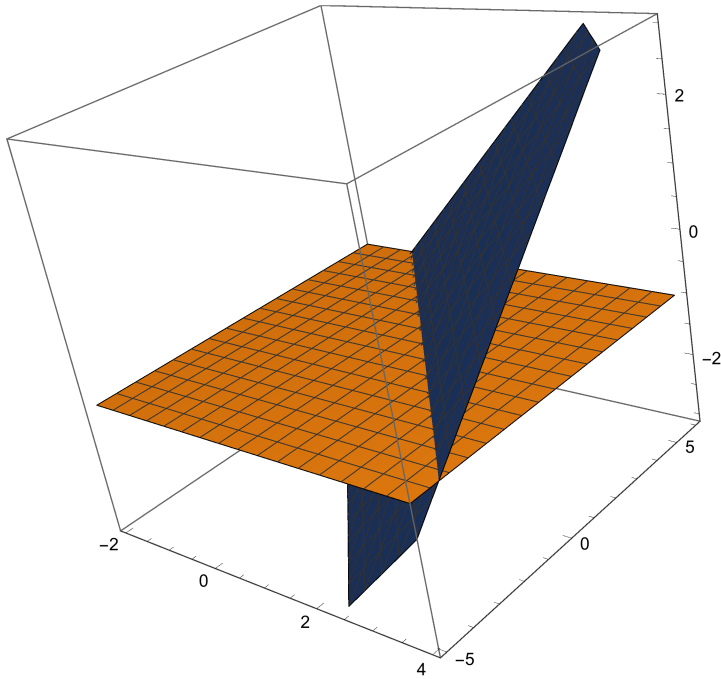
```
ContourPlot3D[ $\left\{2(x-1) + (y-2) - 2(z+1) = 0, x^2 + \left(\frac{y}{2}\right)^2 + z^2 = 3\right\},$   
{x, -2, 4}, {y, -5, 6}, {z, -3, 3}]
```



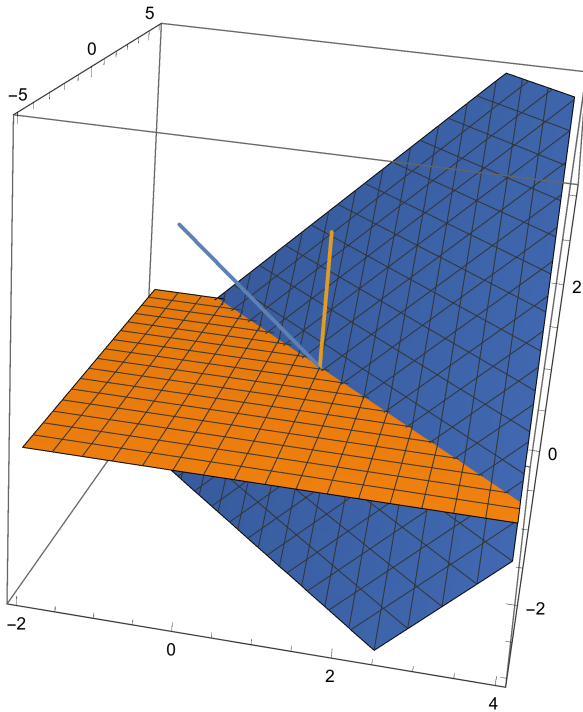
```
ContourPlot3D[{{(x - 1)^2 + ((y - 2)/3)^2 == (z + 1), x^2 + ((y/2)^2 + z^2 == 3, -z == 1,
  2 (x - 1) + (y - 2) - 2 (z + 1) == 0}, {x, -2, 4}, {y, -5, 6}, {z, -3, 3}]
```



```
ContourPlot3D[{-z == 1, 2 (x - 1) + (y - 2) - 2 (z + 1) == 0}, {x, -2, 4}, {y, -5, 6}, {z, -3, 3}]
```

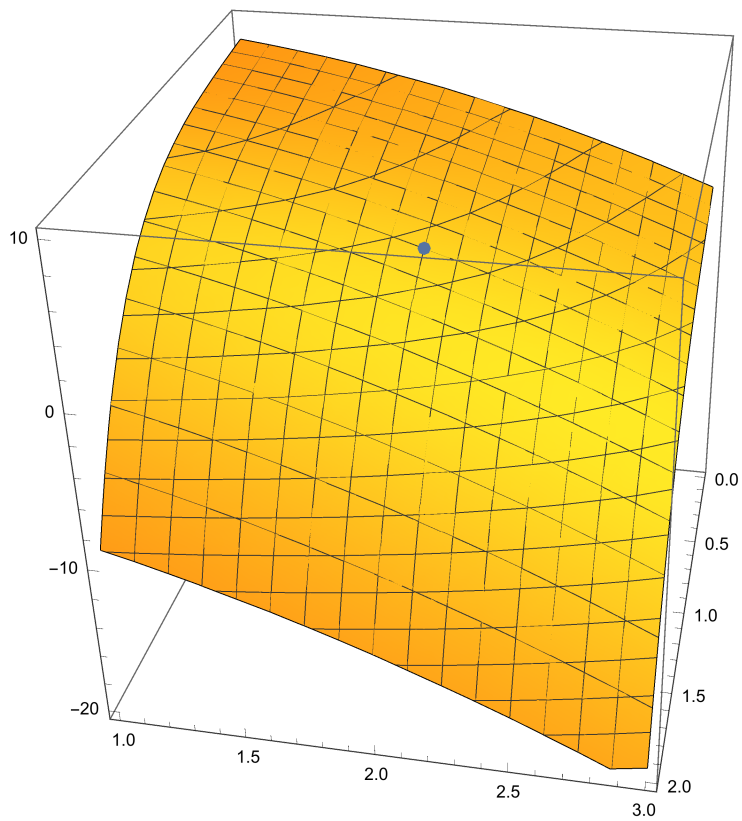


```
Show[ContourPlot3D[{-z == 1, 2 (x - 1) + (y - 2) - 2 (z + 1) == 0}, {x, -2, 4}, {y, -5, 6},
{z, -3, 3}], ParametricPlot3D[{{1 + 2 t, 2 + t, -1 - 2 t}, {1, 2, t}}, {t, -1, 1}]]
```

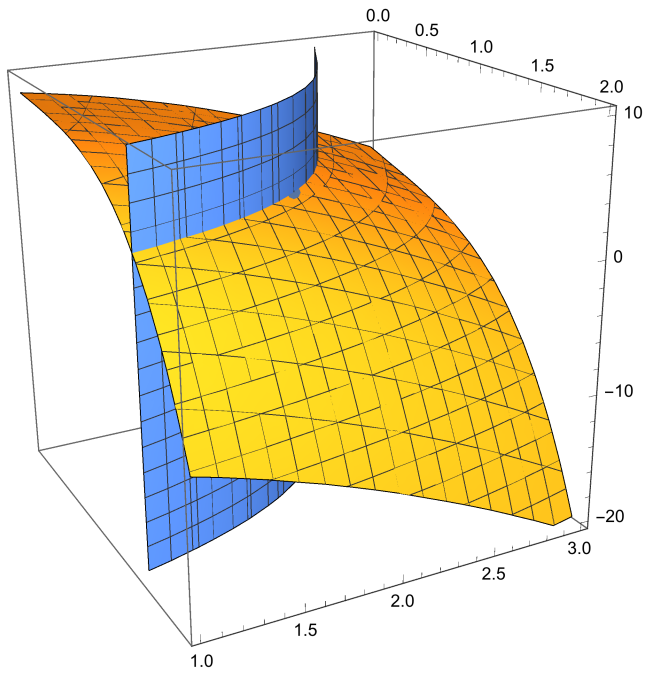


We find the direction of travel that maintains a constant height by using the fact level curves are perpendicular to the gradient.

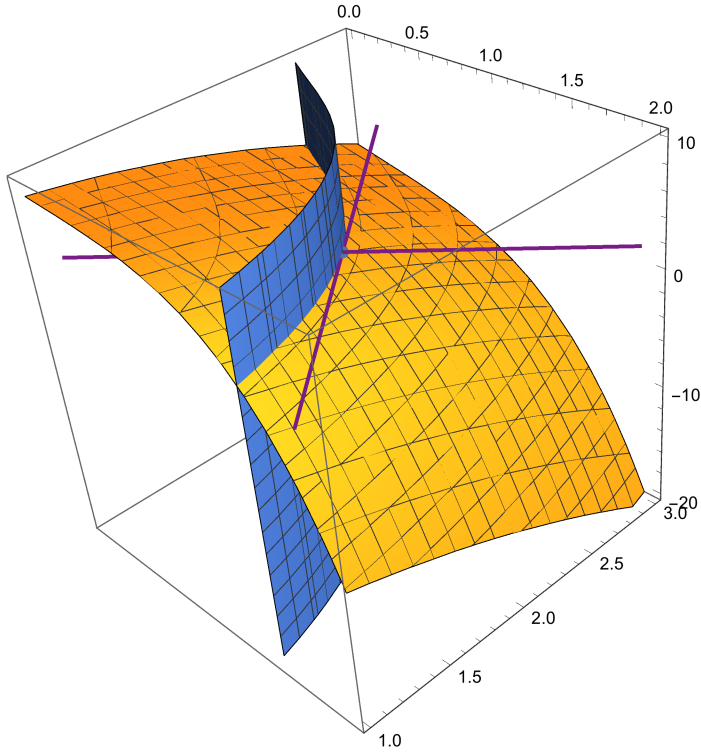
```
Show[ContourPlot3D[z == 10 - x^4 - x y - y^2, {x, 0, 2}, {y, 1, 3}, {z, -20, 10}],  
ListPointPlot3D[{{1, 2, 3}}, Filling -> Bottom,  
PlotStyle -> PointSize[Large], PlotStyle -> Black]]
```



```
Show[ContourPlot3D[{z == 10 - x^4 - x y - y^2, 0 == 10 - x^4 - x y - y^2 - 3},  
  {x, 0, 2}, {y, 1, 3}, {z, -20, 10}], ListPointPlot3D[{{1, 2, 3}},  
  Filling -> Bottom, PlotStyle -> PointSize[Large], PlotStyle -> Black]]
```

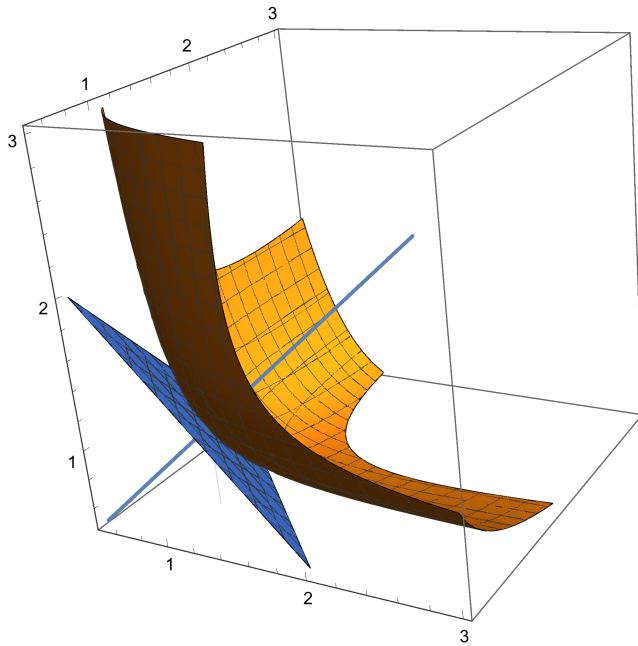


```
Show[ContourPlot3D[{z == 10 - x^4 - x y - y^2, 0 == 10 - x^4 - x y - y^2 - 3},
  {x, 0, 2}, {y, 1, 3}, {z, -20, 10}], ListPointPlot3D[{{1, 2, 3}},
  Filling -> Bottom, PlotStyle -> PointSize[Large], PlotStyle -> Black],
  ParametricPlot3D[{{1 - 6 t, 2 - 5 t, 3}, {1 - 5 t, 2 + 6 t, 3}},
  {t, -2, 2}, ColorFunction -> "Rainbow"]]
```



We compare the meaning of the gradient in the first and last

```
Show[ContourPlot3D[{x^y y^z z^x == 1, x + y + z == 3}, {x, .5, 3}, {y, .5, 3},  
  {z, .5, 3}], ParametricPlot3D[{1 + t, 1 + t, 1 + t}, {t, -.5, 1}], ListPointPlot3D[  
  {{1, 1, 1}}, Filling -> Bottom, PlotStyle -> PointSize[Large], PlotStyle -> Black]
```



```
Show[ContourPlot3D[{z == 10 - x^4 - x y - y^2, -6 (x - 1) + -5 (y - 2) + - (z - 3) == 0},  
  {x, 0, 2}, {y, 1, 3}, {z, -20, 10}], ListPointPlot3D[{{1, 2, 3}},  
  Filling -> Bottom, PlotStyle -> PointSize[Large], PlotStyle -> Black],  
  ParametricPlot3D[{{1 - 6 t, 2 - 5 t, 3}}, {t, -2, 2}, ColorFunction -> "Rainbow"]]
```

