Math 113 Problem Set 9

Due April 10, 2018

April 3, 2018

1. Show that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$. (4.4, #1)
2. Let A and B be regions as in Riemann Mapping Theorem. Given $z_0 \in A$, $w_0 \in B$, and angle θ_0, show that there is a unique conformal equivalence $f : A \to B$ with $f(z_0) = w_0$ and $\arg f'(z_0) = \theta_0$. (5.1, #7)
3. Let $f : A \to B$ be a functions such that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial x}$ exist and are continuous. Suppose that f is a bijection that preserves angles. Prove that f is analytic and conformal. (5.1, #8; there's a hint in the book.)
4. Let $f(z) = \frac{z-i}{z+i}$. What is the image under f or
   (a) the real line
   (b) the circle with center 0 and radius 2
   (c) the circle with center 0 and radius 1
   (d) the imaginary axis

   (5.2, #2)
5. Conformally map the region $\{z \mid \text{Re } z < 0, 0 < \text{Im } z < \pi\}$ onto the first quadrant. (5.2, #25)
6. Conformally map the circle of radius 1 around 1 onto $\{z \mid \text{Re } z > 1\}$. (5.2, #26)
7. Conformally map $\mathbb{C} - \{\text{nonpositive real axis}\}$ onto $\{z \mid -\pi < \text{Im } z < \pi\}$. (5.2, #27)

For problems from the book, something like 1, #8 refers to #8 from the exercises at the end of Chapter 1, while something like 1.3, #1 refers to #1 from the exercises at the end of section 1.3.