

Math 113 Problem Set 8

Due April 3, 2018

March 25, 2018

1. (a) Prove that

$$\int_{-\infty}^{\infty} \frac{\cos x}{e^x + e^{-x}} dx = \frac{\pi}{e^{\pi/2} + e^{-\pi/2}}$$

by integrating around the contour mentioned in the book for this problem.

- (b) In the same way, show

$$\int_{-\infty}^{\infty} \frac{e^{-x}}{1 + e^{-2\pi x}} dx = \frac{1}{2 \sin \frac{1}{2}}$$

(4.3, #8)

2. Find $\int_0^1 \frac{1}{\sqrt{x^2-1}} dx$ by following the method outlined in the book for this problem. (4.3, #15)
3. Evaluate $\int_0^\pi \frac{d\theta}{2 \cos \theta + 3}$. (4, # 7)
4. Evaluate $\int_0^{2\pi} \frac{\sin^2 x}{5+3 \cos x} dx$.
5. Evaluate $\int_0^\infty \frac{x^2}{(x^2+4)^2(x^2+9)} dx$.
6. Compute the principal value of $\int_{-\infty}^\infty \frac{\sin x}{x(x+1)(x^2+1)} dx$. (4, #5)
7. Evaluate $\int_{|z|=1} \frac{\cos(e^{-z})}{z^2} dz$. (4, #11)
8. Let f have a zero of order k at z_0 . Show that $\text{Res}(\frac{f'}{f}; z_0) = k$ and find the residues of $\frac{f''}{f'}$ and $\frac{f''}{f}$ at z_0 . (4, #19)
9. Compute $\int_{-\infty}^\infty \frac{1}{x^{2n}+1} dx$.

For problems from the book, something like 1, #8 refers to #8 from the exercises at the end of Chapter 1, while something like 1.3, #1 refers to #1 from the exercises at the end of section 1.3.