

Math 113 Problem Set 5

Due Feb. 27, 2018

February 20, 2018

1. Define the function $f : \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} 0 & x \leq 0 \\ e^{-\frac{1}{x}} & x > 0 \end{cases}$$

Show that f is C^∞ (infinitely differentiable), but has no power series representation valid on all of \mathbb{R} .

2. (a) Show that $\sum_{n^2+z^2}^{\infty} \frac{1}{n^2+z^2}$ converges on $\mathbb{C} - \{ni \mid n \in \mathbb{Z}\}$.
(b) Show that this convergence is uniform and absolute on each closed disk in $\mathbb{C} - \{ni \mid n \in \mathbb{Z}\}$. (3.1, #4)
3. (a) Show that z^n converges uniformly to the zero function on each $B_{\leq r}(0)$ for $r < 1$.
(b) Is the convergence uniform on $B_{\leq 1}(0)$? (3.1, #5)
4. Prove that $\zeta(z) = \sum_{n^z}^{\infty} \frac{1}{n^z}$ does not converge uniformly on $\{z \mid \operatorname{Re} z > 1\}$. (3.1, #8)
5. Let $f : B_2(0) \rightarrow B_{\leq 7}(0)$ be analytic. Prove that there is a $\delta > 0$ such that if $z, w \in B_1(0)$ with $|z - w| < \delta$, then $|f(z) - f(w)| < \frac{1}{10}$. Find a numerical value for δ independent of f . (3.1, #16)
6. Find a region where $\sum_{n}^{\infty} \frac{(2z-1)^n}{n}$ is analytic. (3.1, #19)

For problems from the book, something like 1, #8 refers to #8 from the exercises at the end of Chapter 1, while something like 1.3, #1 refers to #1 from the exercises at the end of section 1.3.