Math 113 Problem Set 4

Due Feb. 20, 2018

February 12, 2018

1. Let $\gamma : [a, b] \to \mathbb{C}$ be a piecewise smooth curve. Show that $I(\gamma; \cdot)$ is a continuous function on $\mathbb{C} - \gamma$; that is, if $z_0 \in \mathbb{C}$ is not on the curve γ, then $I(\gamma; \cdot)$ is continuous at z_0.

2. Liouville's Theorem can be refined to be much more precise. For instance, set $f(z) = |z| \cos |z|$. We suspect this is not analytic since $|z|$ is not. Show that f is not analytic using a "Liouville-style" argument.

3. 2.4, #13, this has you evaluate several integral using Cauchy's Integral Formula

4. Find harmonic conjugates of the following (and specify a region)

   (a) $\sinh x \sin y$

   (b) $\log \sqrt{x^2 + y^2}$

   (c) $e^x \cos y$

   (2.5, #9)

5. Let $0 < r_1 < r_2 < r_3$ and $R = \{z \in \mathbb{C} \mid r_1 < z < r_3\}$. Let M_ℓ be the maximum of $|f|$ on $|z| = r_\ell$ for $\ell = 1, 2, 3$. Show that

$$M_2^{\log \frac{r_3}{r_1}} \leq M_1^{\log \frac{r_3}{r_2}} M_3^{\log \frac{r_2}{r_1}}$$

   (2.5, #14)

6. Let f be analytic on and inside the unit circle. Suppose that the image of $|z| = 1$ lies in $B_r(z_0)$. Show that the image of $B_{\leq 1}(0)$ lies in $B_r(z_0)$. Illustrate with $f(z) = e^z$.(2, #16)

For problems from the book, something like 1, #8 refers to #8 from the exercises at the end of Chapter 1, while something like 1.3, #1 refers to #1 from the exercises at the end of section 1.3.