Math 113 Problem Set 2

Due Feb. 13, 2018

February 6, 2018

1. Let γ_1 be the unit circle and γ_2 be the circle of radius 2 centered at the origin (both traversed once counterclockwise). Show that

$$\int_{\gamma_1} \frac{dz}{z^3(z^2+10)} = \int_{\gamma_2} \frac{dz}{z^3(z^2+10)}$$

   (2.2, #8)

2. Prove that $\mathbb{C} - \{0\}$ is not simply connected. (2.3, #1)

3. Show that a star-shaped region is simply connected. (2.3, # 3, includes definition of star-shaped)

4. 2.3, # 7. This asks you to evaluate several integrals without an explicit calculation.

5. Show that the computation of the arclength of a curve is independent of the parameterization.

6. In the proof of Cauchy's Theorem for a Disk (Theorem 2.3.2), show that

$$F(z) + \int_{\langle\langle z,w \rangle\rangle} f(\alpha)d\alpha = F(w)$$

7. Show that the relation 'γ_1 homotopic to γ_2' is an equivalence relation on closed curves.

8. Evaluate the following (without performing the explicit computation):

   (a) $\int_\gamma \frac{z^2}{z-1}dz$ where γ is the circle of radius 2 centered at 0

   (b) $\int_\gamma \frac{e^z}{z^2}dz$ where γ is the unit circle.

   (2.4, #1)

9. Find the maximum of $|e^z|$ on $|z| \leq 1$. (2.5, #1)

For problems from the book, something like 1, #8 refers to #8 from the exercises at the end of Chapter 1, while something like 1.3, #1 refers to #1 from the exercises at the end of section 1.3.