

# Math 113 Problem Set 1

Due Jan. 30, 2018

January 23, 2018

1. What is the complex conjugate of  $\frac{(8-2i)^{10}}{(4+6i)^5}$ ? (1.2, #4)
2. Let  $\sqrt{\cdot}$  denote the particular square root defined by

$$\sqrt{r(\cos \theta + i \sin \theta)} = r^{\frac{1}{2}} \left( \cos \frac{\theta}{2} + i \sin \frac{\theta}{2} \right), \text{ for } 0 \leq \theta < 2\pi$$

For what values of  $z$  does  $\sqrt{z^2} = z$ ? (1.3, #10)

3. Show that  $f(z) = \bar{z}$  is continuous. (1.4, #7)
4. Define the chordal metric  $\rho$  on  $\bar{\mathbb{C}}$  by setting

$$\rho(z_1, z_2) = d(z'_1, z'_2)$$

where  $z'_\ell$  is the point on the Riemann sphere corresponding to  $z_\ell$  and  $d$  is the usual distance in  $\mathbb{R}^3$ .

- (a) Show that  $z_n \rightarrow z$  in  $\mathbb{C}$  iff  $\rho(z_n, z) \rightarrow 0$ .
- (b) Show that  $z_n \rightarrow \infty$  iff  $\rho(z_n, \infty) \rightarrow 0$ .
- (c) If  $f(z) = \frac{az+b}{cz+d}$  and  $ad - bc \neq 0$ , show that  $f$  is continuous at  $\infty$ .

(1.4 #24)

5. Describe geometrically the set of points  $z \in \mathbb{C}$  satisfying

- (a)  $|z + i| = |z - i|$
- (b)  $|z - 1| = 3|z - 2|$

(1, #7)

6. Can is single-valued (analytic) branch of  $\log z$  be defined on the following sets?

- (a)  $\{z \mid 1 < |z| < 2\}$
- (b)  $\{z \mid \operatorname{Re} z > 0\}$
- (c)  $\{z \mid \operatorname{Re} z > \operatorname{Im} z\}$

(1, #13)

7. Show that the map  $z \mapsto z + \frac{1}{z}$  maps the circle  $\{z \mid |z| = c\}$  onto the ellipse described by

$$\left\{ z = x + iy \mid x = \left( c + \frac{1}{c} \right) \cos \theta, y = \left( c - \frac{1}{c} \right) \sin \theta \text{ for } 0 \leq \theta \leq 2\pi \right\}$$

(1, #14)

8. Prove that

$$\lim_{h \rightarrow 0} \frac{(z_0 + h)^n - z_0^n}{h} = n z_0^{n-1}$$

for any  $z_0 \in \mathbb{C}$ . (1, #23)

9. Use de Moivre's theorem to find the sum

$$\sin x + \sin 2x + \cdots + \sin nx$$

(1, #32)

10. Determine the sets on which the following functions are analytic and compute their derivatives:

(a)  $(z + 1)^3$

(b)  $z + \frac{1}{z}$

(c)  $\left(\frac{1}{z-1}\right)^{10}$

(d)  $\frac{1}{(z^3-1)(z^2+2)}$

(1.5, #1)

11. Is it true that  $|\sin z| \leq 1$  for all  $z \in \mathbb{C}$ ? (1.6, #5)

12. Evaluate  $\int_{\gamma} y dz$ , where  $\gamma$  is the union of the line segments joining 0 to  $i$  and then to  $i + 2$ . (2.1, #1(a))

Note that I have written out the problems that appear in Marsden and Hoffman because it's still shopping period. You shouldn't rely on this in future problem sets.