MATH 122 PSET 5 DUE 10/16 BY THE END OF THE DAY

The total number of points for all problems is 105. The maximum score you can earn on this problem set is 100. Therefore, if you solve problems worth a total of x points, your score will be $\min(x, 100)$.

I remind you that using AI to give you answers to or help you answer homework problems is just as much cheating and unethical and honor code violating as asking a person to do that. I trust you will hold yourself to the highest ethical standards!

1. PROBLEM 1

(a) (5 points) Compute the order of each of the elements in D_{10}.
(b) (15 points) Show that every element of D_{2n} which is not a power of r has order 2.

2. PROBLEM 2

(a) (15 points) If $n = 2k$ is even and $n \geqslant 4$, show that $z = r^k$ is an element of order 2 which commutes with all elements of D_{2n}.
(b) (15 points) Show that z is the only nonidentity element of D_{2n} that commutes with all elements of D_{2n}.

3. PROBLEM 3

(a) (5 points) Show that D_{2n} is generated by two elements s and sr.
(b) (15 points) We will prove in class that:
$$D_{2n} = \langle s, r \rangle / \langle s^2 = r^n = 1, \ rs = sr^{-1} \rangle. \tag{3.1}$$

Set
$$a = s, \ b = sr,$$
show that
$$D_{2n} = \langle a, b \rangle / \langle a^2 = b^2 = (ab)^n = 1 \rangle. \tag{3.2}$$

Hint: use that relations in (3.1) imply the relations in (3.2) and vice versa.

4. PROBLEM 4

(a) (10 points) Prove that for every divisor $d | n$ the cyclic subgroup $\langle r^d \rangle \subset D_{2n}$ is normal.
(b) (10 points) Describe the quotient group $D_{2n} / \langle r^d \rangle$.

Hint: show that it is isomorphic to D_{2d}

(c) (10 points) If $n = 2k$ is even, compute the orders of the subgroups $\langle r^2, s \rangle$, $\langle r^2, sr \rangle \subset D_{2n}$ and prove that they are normal.

(d)* (5 points) This problem is more complicated! Prove that any normal subgroup of D_{2n} is 1, D_{2n} or as in (a), (c) above.