Math 136: Homework 7

Due: Friday, December 1

1. The Lorentz, or Minkowski metric on $\mathbb{R}^{3,1} = \mathbb{R}^4$ is
$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2.$$
Show that this metric is invariant under rotations in the (x, y, z) plane, and *boosts*. A boost along the x-axis is a change of coordinates of the form
$$t' = \gamma\left(t - \frac{vx}{c^2}\right), \quad x' = \gamma(x - vt), \quad y' = y, \quad z' = z$$
where v is a constant, and $\frac{1}{\gamma} = (\sqrt{1 - \frac{v^2}{c^2}})$. Suppose you are standing at the origin in space time. Explain why these "boosted" coordinates correspond to the coordinates of another observer moving relative to you at speed v along the x-axis.

2. Suppose that you are standing at the origin in space with a flash light, and your best friend is moving along the x-axis at speed $v = \frac{c}{\sqrt{2}}$ where c is the speed of light. You turn on your flash light for one second. According to your friend, how long was the flashlight on for?

3. Recall the Schwarzchild metric of general relativity.
$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2(d\phi^2 + \sin^2(\phi) d\theta^2)$$
Notice that it "looks like" the metric becomes singular at the Schwarzchild radius $r = 2GM$, in the sense that the metric fails to be invertible. This problem shows that this is actually just a problem with the *coordinates* and not the metric. Consider the "tortoise" coordinate
$$r^* = r + 2GM \log\left|\frac{r}{2GM} - 1\right|.$$
Compute dr^*/dr. Change coordinates by replacing t with $v = t + r^*$. These are called the *ingoing* Eddington-Finkelstein coordinates. Show that in these coordinates the metric is given by
$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dv^2 + 2dvdr + r^2(d\phi^2 + \sin^2(\phi) d\theta^2)$$
Note that in these coordinates, the metric is still invertible at $r = 2GM$!