

# Math 136: Homework 6

Due: Friday, November 3

1. Suppose that  $S$  is a smooth surface. Let  $(u, v)$  be local coordinates for  $S$ . Compute the connection, and Gaussian curvature of  $S$ , in the following situations:
  - (i)  $g(u, v) = \lambda(u, v)(du^2 + dv^2)$  for  $\lambda(u, v) > 0$ .
  - (ii)  $g(u, v) = du^2 + \lambda(u, v)dv^2$  for  $\lambda(u, v) > 0$ .

2. If  $f : U \rightarrow S \subset \mathbb{R}^3$  is a parametrized surface, show that the Christoffel symbols are given by

$$\Gamma_{ij}^k = g^{km} \left\langle \frac{\partial^2 f}{\partial u_i \partial u_j}, \frac{\partial f}{\partial u_m} \right\rangle.$$

Note that this formula is in general, much simpler to compute than the intrinsic formula using the metric.

3. Kühnel, Chapter 4, problem 14
4. Kühnel, Chapter 4, problem 15
5. Kühnel, Chapter 4, problem 16