Math 136: Differential Geometry– Homework 4

Due Wednesday, October 13, 2017

(1) Let $c : I \to \mathbb{R}^3$ be a curve, and $F : U \to \mathbb{R}^3$ be a surface. Suppose that $c(I) \subset S$.

(i) Show that the curvature of c satisfies

$$\mathbf{K} = \sqrt{\kappa_g^2 + \kappa_\nu^2}$$

where κ_g is the geodesic curvature and κ_ν is the normal curvature.

(ii) Suppose that S has positive Gaussian curvature (ie. $\det L > 0$, where L denotes the Weingarten map). Show that the curvature $\mathbf{K}$ of C at p satisfies

$$\mathbf{K} \geq \min\{||\kappa_1|, |\kappa_2|\} \tag{1}$$

where κ_1, κ_2 denote the principal curvatures (ie. eigenvalues of L) of S at p.

(2) Kühnel, Chapter 3, Problem 16 (page 130).

(3) Consider the surface of revolution obtained by rotating the curve $x = f(z)$ around the (x, z)- plane. This surface is parametrized by

$$(z, \theta) \mapsto (f(z)\cos(\theta), f(z)\sin(\theta), z). \tag{2}$$

Show that the inward mean curvature is given by

$$H = \frac{\kappa}{2} + \frac{1}{2f\sqrt{1 + (f')^2}} = \frac{\kappa}{2} + \frac{\cos\theta}{2f}, \tag{3}$$

where

$$\kappa = -\frac{f''}{(1 + (f')^2)^{3/2}} \tag{4}$$

is the inward curvature of the curve $(f(z), z)$, and θ is the angle of the curve with the z-axis. Check that this formula yields the correct answer for the sphere of radius 1 centered at the origin.

(4) Using problem 3, compute the mean curvature for the catenoid, given by $f(z) = \cosh(z)$. (**BONUS**) Show that the catenoid and the plane are the only surfaces of revolution which are also minimal surfaces.

(5) Find the principal curvatures and the principal lines of curvature for the catenoid.