Math 136: Differential Geometry– Homework 3

Due Friday, September 29, 2017

(1) The Möbius strip (see figure 3.5 in Kühnel) is parametrized by the map

$$f(u,v) = \Big(\sin(u) + v\sin(\frac{u}{2})\sin(u), \cos(u) + v\sin(\frac{u}{2})\cos(u), v\cos(\frac{u}{2})\Big) \tag{1}$$

where $(u,v) \in [0,2\pi] \times (-1,1)$.

(i) Show that $f(0,v) = f(2\pi,-v)$. In particular, $f(0,0) = f(2\pi,0)$.

(ii) Show that there is no normal vector field ν of unit length on the Möbius strip which satisfies $\nu(0,0) = \nu(2\pi,0)$. (Hint: Compute $\frac{\partial f}{\partial u} \times \frac{\partial f}{\partial v}$ at the points $(0,0)$ and $(2\pi,0)$. Now argue by contradiction.)

(2) The catenoid is described parametrically by

$$f(x,y) = (\cosh x \cos y, \cosh x \sin y, x), \qquad (x,y) \in (-\infty,\infty) \times [0,2\pi] \tag{2}$$

(i) Draw a picture of a segment of the catenoid.

(ii) Find the metric on the catenoid.

(iii) Let $\Sigma_t := (-t,t) \times [0,2\pi]$. Find the image of the Gauss map $\nu(\Sigma_t)$, and describe the set $\{p \in S^2 : \forall t,\ p \notin \nu(\Sigma_t)\}$.

(iv) Find the Weingarten map (or Shape Operator) of the catenoid.

(3) Prove the following statement. Let S be an oriented regular surface, and let $\alpha : S \to \mathbb{R}$ be a smooth function with compact support. Then the integral

$$\int_S \alpha dA \tag{3}$$

is independent of oriented parametrization.

(4) Show that the Gauss map is independent of the choice of oriented parametrization.