Math 136: Differential Geometry– Homework 2

Due Friday, September 22, 2015

(1) Kühnel, Chapter 2, Problem 9 (page 50 in Vol. 2, page 51 in Vol. 3)

(2) Kühnel, Chapter 2, Problem 10 (page 50 in Vol. 2, page 51 in Vol. 3)

(3) Kühnel, Chapter 2, Problem 11 (page 51)

(4) Kühnel, Chapter 3, Problem 1 (page 127)

(5) Let $S \subset \mathbb{R}^3$ be a local, regular parametrized surface. Let g be the metric on S induced by restriction of the standard inner product on $\mathbb{R}^3$. Suppose that $p \in S$, and that $X, Y \in T_pS$ are two tangent vectors. Show that the *number* $g(X, Y)$ is independent of the choice of parametrization.

(6) A surface $S \subset \mathbb{R}^3$ is called a surface of rotation if there is a regular plane curve $t \mapsto (r(t), h(t)) \in \mathbb{R}^2$ such that S is obtained by rotating this curve around the z axis. In other words, S can be parametrized by $(t, \theta) \mapsto (r(t)\cos\theta, r(t)\sin\theta, h(t))$. Compute the metric (first fundamental form) of S.