Math 112: Real Analysis– Homework 6

Due Tuesday, March 22, 2016

(1) Rudin, Chapter 3, Problems 3, 6 (a)-(c), 7

(2) Suppose that $\{a_n\}$ is a sequence of points in a metric space (X, d), such that

$$d(a_n, a_{n+1}) < c^2 d(a_n, a_{n-1}).$$

for some $c \in (0, 1)$. Show that $\{a_n\}$ is a Cauchy sequence. (**Fun Fact**: This little problem is a key step in the proof of the implicit function theorem).

(3) Let $\{a_n\}$ and $\{b_n\}$ be two Cauchy sequences in (X, d). Show that $d(a_n, b_n)$ is a Cauchy sequence in $\mathbb{R}$. (**Fun Fact**: This little problem is used in one method for constructing the real numbers).

(4) Suppose that $\{a_n\}$ is a Cauchy sequence in (X, d), and suppose that some subsequence $\{a_{n_k}\}$ converges to a point $p \in X$. Show that $\{a_n\}$ converges to p.