MATH 99r: Presentation - Cana Nakaye

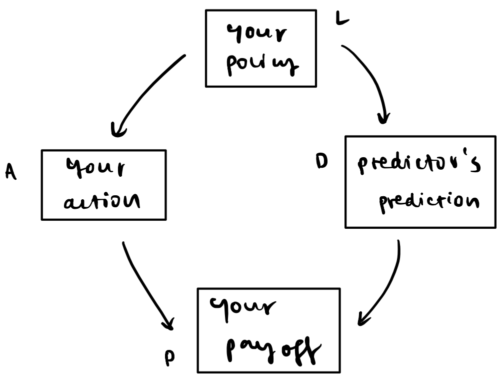
GOAL: EDT and CDT on Newcomb's Problem

Recall:

Newcomb's Problem

- Predictor with high accuracy (99%)
- Another player
- two boxes A and B
- Given a choice between
  - only taking B or
  - both A and B.
- Box A: $1000 | Box B: $ 1M if predictor predicts only taking B; $ 0 " taking A and B
- The player does not know what the predictor predicted or what box B contains while making the choice

Bayes net for this problem



policy: 1-box	policy: 2-box
.5	.5

← we're choosing to model the a priori prob. of 1 box vs. 2 box as 50:50

action

policy	1-box	2-box
1-box	1	0
2-box	0	1

prediction

policy	1-box	2-box
1-box	.99	.01
2-box	.01	.99

payoff

action	prediction	$0	$1000	$1M	$1M + 1000
1-box	1-box	0	0	1	0
1-box	2-box	1	0	0	0
2-box	1-box	0	0	0	1
2-box	2-box	0	1	0	0

EDT on decision node A:

We want to compute $E[P \mid A = 1\text{-box}]$, $E[P \mid A = 2\text{-box}]$

these formula:

$$\Pr(p \mid a) = \sum_{\ell, d} \Pr(p, \ell, d \mid a)$$
$$= \sum_{\ell, d} \Pr(\ell \mid a) \Pr(d \mid \ell, a) \Pr(p \mid \ell, d, a)$$
$$= \sum_{\ell, d} \underline{\Pr(\ell \mid a)}\ \underline{\Pr(d \mid \ell)}\ \underline{\Pr(p \mid d, a)}$$

From the prob. tables, we have values for $\Pr(d \mid \ell)$ and $\Pr(p \mid d, a)$.

For the red term, consider Bayes' rule: $\Pr(\ell \mid a) = \dfrac{\Pr(a \mid \ell)\Pr(\ell)}{\Pr(a)}$.

when $L = $ 1-box, $A = $ 1-box, $\Pr(\ell \mid a) = \dfrac{\Pr(a \mid \ell)\Pr(\ell)}{\Pr(a=1 \mid L=1)\Pr(L=1) + \Pr(a=1 \mid L=2)\Pr(L=2)}$

$$= \frac{1 \cdot .5}{1 \cdot .5 + 0 \cdot .5} = 1$$

$L = $ 2-box, $A = $ 1-box, $\Pr(\ell \mid a) = \dfrac{0 \cdot .5}{0.5} = 0$

$L = $ 1-box, $A = $ 2-box $\Pr(\ell \mid a) = \dfrac{0 \cdot .5}{0.5} = 0$

$L = $ 2-box, $A = $ 2-box $\Pr(\ell \mid a) = \dfrac{1 \cdot .5}{0.5} = 1$

	policy	
action	1-box	2-box
1-box	1	0
2-box	0	1

combining these together,

	payoffs			
action	\$0	\$1000	\$1M	\$1M + 1000
1-box	.01	0	.99	0
2-box	0	.99	0	0.01

$\Pr(P = 1M \mid A = 1)$
$= \Pr(L=1 \mid A=1)\Pr(D=1 \mid L=1) + \Pr(L=2 \mid A=1)\Pr(D=1 \mid L=2)$
$= 1 \cdot 0.99 + 0 \cdot 0.01$
$= 0.99$

$\Pr(P = 0 \mid A = 1)$
$= \Pr(L=1 \mid A=1)\Pr(D=2 \mid L=1) + \Pr(L=2 \mid A=1)\Pr(D=2 \mid L=2)$
$= 1 \cdot 0.01 + 0 \cdot 0.99$
$= 0.01$

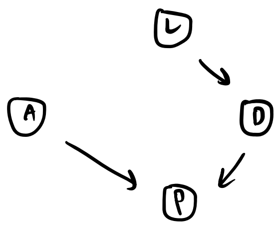
$Pr(P=1000 \mid A=2)$
$= Pr(L=1 \mid A=2)\, Pr(D=2 \mid L=1) + Pr(L=2 \mid A=2)\, Pr(D=2 \mid L=2)$
$= 0 \cdot 0.01 + 1 \cdot 0.99$
$= 0.99$

$Pr(P=1M+1000 \mid A=2)$
$= Pr(L=1 \mid A=2)\, Pr(D=1 \mid L=1) + Pr(L=2 \mid A=2)\, Pr(D=1 \mid L=2)$
$= 0 \cdot 0.99 + 1 \cdot 0.01$
$= 0.01$

Therefore,
$E[P \mid a=1\text{-box}] = 0 \cdot .01 + \$1M \cdot .99 = \$.99M$
$E[P \mid a=2\text{-box}] = \$1000 \cdot .99 + (\$1M+1000) \cdot .01 = \$990 + \$10010 = \$11000$

$\Rightarrow$ the EDT will choose to 1-box.

CDT on decision node A



We will use a modified Bayes net to compute $E[P \mid do(a)]$ for $a = 1\text{-box}, 2\text{-box}$.

use formula:
$$Pr(P \mid do(a)) = \sum_{l,d} Pr(l)\, Pr(d \mid l)\, Pr(p \mid d, a).$$

All of the terms can be looked up in the tables.

Payoffs

Action	0	\$1000	\$1M	\$1M+1000
1-box	0.5	0	0.5	0
2-box	0	0.5	0	0.5

$Pr(P=0 \mid do(a=1)) = Pr(L=1)\, Pr(d=2 \mid L=1) + Pr(L=2)\, Pr(d=2 \mid L=2)$
$= 0.5 \cdot 0.01 + 0.5 \cdot 0.99$
$= 0.5$

$$\Pr(P=1M \mid do(a=1)) = \Pr(L=1)\Pr(d=1 \mid L=1) + \Pr(L=2)\Pr(d=1 \mid L=2)$$
$$= 0.5 \cdot 0.99 + 0.5 \cdot 0.01$$
$$= 0.5$$

$$\Pr(P=1000 \mid do(a=2)) = \Pr(L=1)\Pr(d=2 \mid L=1) + \Pr(L=2)\Pr(d=2 \mid L=2)$$
$$= 0.5 \cdot 0.01 + 0.5 \cdot 0.99$$
$$= 0.5$$

$$\Pr(P=1M+1000 \mid do(a=2)) = \Pr(L=1)\Pr(d=1 \mid L=1) + \Pr(L=2)\Pr(d=2 \mid L=2)$$
$$= 0.5 \cdot 0.99 + 0.5 \cdot 0.01$$
$$= 0.5$$

Therefore,

$E[P \mid do(a=1\text{-box})] = 0.5 \cdot 0 + 0.5 \cdot 1M = 0.5M$

$E[P \mid do(a=2\text{-box})] = 0.5 \cdot 1000 + 0.5 \cdot (1M + 1000) = 501{,}000$

$\Rightarrow$ The CDT on decision node A will choose 2-box.

Remark. Taking A to be the decision node is funky $\rightarrow$ we're imagining that some process other than "your policy" (which the predictor can't predict) is determining your decision.

CDT (= EDT) on decision node L

CDT is the same as EDT with decision node L, since L has no parents, the do-operator doesn't delete any arrows.

want to compute $E[P \mid \ell]$ for ℓ = 1-box, 2-box

use formula:

$$\Pr(P \mid \ell) = \sum_{a,d} \Pr(P, a, d \mid \ell)$$
$$= \sum_{a,d} \Pr(a \mid \ell)\Pr(d \mid \ell)\Pr(P \mid a, d)$$

All the terms can be looked up in tables.

Payoffs

Policy	0	\$1000	\$1M	\$1M + 1000
1-box	0.01	0	.99	0
2-box	0	0.99	0	0.01

Note that the payoff 0 is only attainable when $a=1, d=2$.

\$1000 — $a=2, d=2$.

\$1M — $a=1, d=1$.

\$1M + 1000 — $a=2, d=1$.

$$Pr(p=0 \mid l=1) = Pr(a=1 \mid l=1)\ Pr(d=2 \mid l=1)\ Pr(p=0 \mid a=1, d=2)$$
$$= 1 \cdot 0.01 \cdot 1 = 0.01$$

$$Pr(p=1000 \mid l=2) = Pr(a=2 \mid l=2)\ Pr(d=2 \mid l=2)\ Pr(p=1000 \mid a=2, d=2)$$
$$= 1 \cdot 0.99 \cdot 1 = 0.99$$

$$Pr(p=1M \mid l=1) = Pr(a=1 \mid l=1)\ Pr(d=1 \mid l=1)\ Pr(p=1M \mid a=1, d=1)$$
$$= 1 \cdot 0.99 \cdot 1 = 0.99$$

$$Pr(p=1M+1000 \mid l=2) = Pr(a=2 \mid l=2)\ Pr(d=1 \mid l=2)\ Pr(p=1M+1000 \mid a=2, d=1)$$
$$= 1 \cdot 0.01 \cdot 1 = 0.01$$

therefore,

$$E[P \mid l=1\text{-box}] = 0 \cdot 0.01 + \$1M \cdot 0.99 = 0.99\ M$$
$$E[P \mid l=2\text{-box}] = \$1000 \cdot 0.99 + \$1M+1000 \cdot 0.01 = \$11,000$$

$\Rightarrow$ The CDT and EDT on decision node L will choose 1-box.