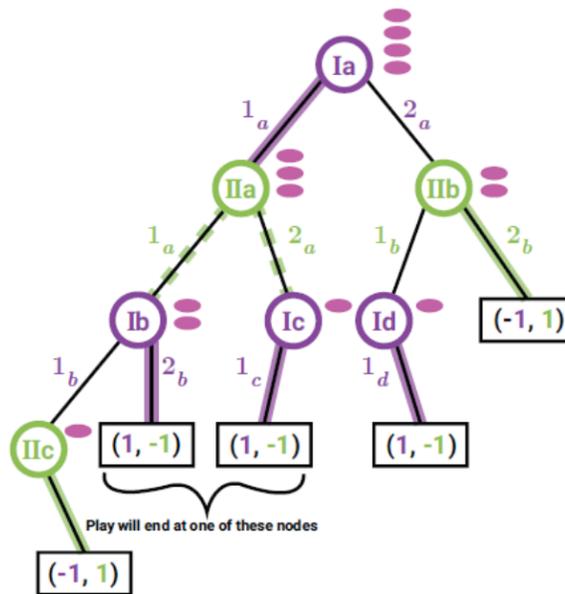


§1, February 16, 2023

—GAMES IN EXTENSIVE FORM—

We will start by playing the subtraction game. Here’s how the game works: We start with four chips (or chocolates). Two players alternate between taking one or two chips at a time. Player 1 goes first. The player who removes the last chip wins.

To better understand the game and its strategies, we can use the following *game tree* from Karlin and Peres’s textbook:



Definition 1.1: A *game tree* is a representation of how a game unfolds with each leaf being labeled with the payoffs of the players and nodes representing possible outcomes.

Say you are player 1. How could you find the winning strategy? We can use what Karlin and Peres called “backward induction, that is, determine what action to play from the leaves up.”

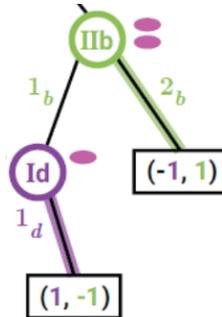
If you want to find the best strategy:

1. Take note of the final nodes where player one wins. In this case there are three.
2. Then continue down the nodes that show the highest payoffs.
3. Note that two of them stem from a single player 2 action, which means you will be guaranteed to win by taking one coin instead of 2.

Within each game, at each node, we also can have *subgames*.

Definition 1.2: *Subgames* are the game that would result if play started at a particular node.

For example, this would a subgame within the subtraction game:



Games can also have subgame-perfect equilibrium. Note, that this is sometimes referred to as “subgame perfect Nash equilibrium.”

Definition 1.3: *Subgame-perfect equilibrium* are the games where the behaviour of each subgame in the game is an equilibrium.

Definition 1.4: An *(Nash) Equilibrium* in a two player game would be a strategy such that neither player could deviate from the strategy and increase their payoff.

We can check subgame-perfect equilibria using backward induction.

This game is an example of an *extensive-form game*. For other finite and extensive-form games, the same backward induction technique can be applied to compute any player’s best strategy in a finite game tree by following the same steps as before.

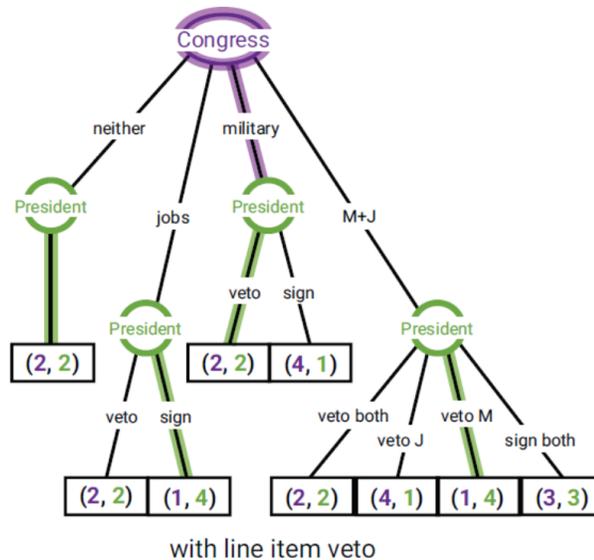
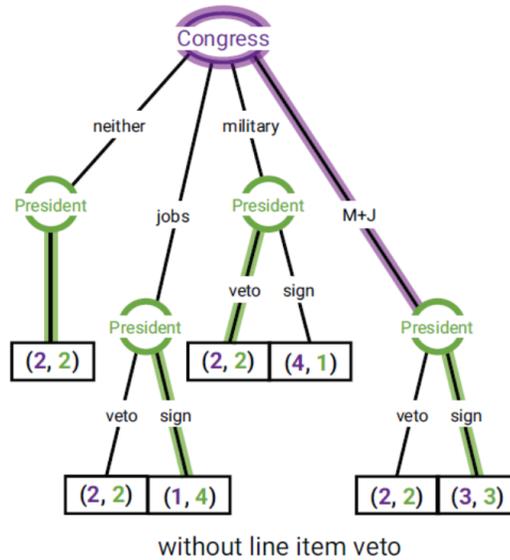
—EXAMPLES—

Line-item veto:

Congress and the President are at odds over spending. Congress prefers to increase military spending (M), whereas the President prefers a jobs package (J). However, both prefer a package that includes military spending and jobs to a package that includes neither. The president can veto specific lines in legislation that they dislike, hence why this example is referred to as the line-veto example, or veto the complete legislation. The payoffs are summarized in the following table:

	Military	Jobs	Both	Neither
Congress	4	1	3	2
President	1	4	3	2

The game trees are as follows:



The highlighted edges show the actions taken in the subgame-perfect equilibrium. *Exercise 1.5:* Would the president would be better off with or without the ability to perform a line item veto?

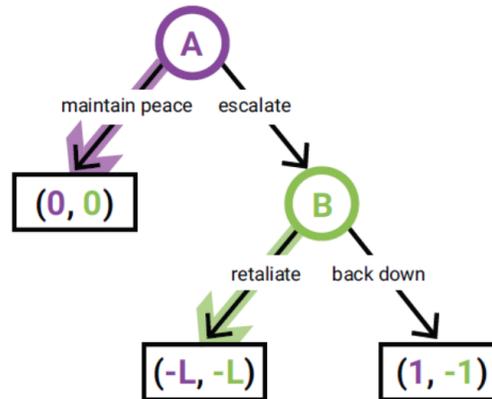
With the line-item veto, the result will be military spending, whereas without the line-item veto, the result will be a military and jobs bill.

Now, we'll see an example of a game that does not have subgame-perfect equilibria.

Mutually assured destruction (MAD):

In this game we have two countries: the Kingdom of B6 and the Principality of DAL, each have nuclear weapons. B6 is aggressive and DAL is benign. B6 may choose between two options. The first is to escalate the arms race, e.g., by firing test missiles, attacking a neighboring country, etc. The second is to do nothing and simply maintain the peace. If B6 escalates, then DAL has two options: retaliate, or back down.

The game trees are as follows:



Exercise 1.6: Why is (escalate, back down) is the only subgame perfect equilibrium?

Objectives:

1. Draw a **game tree** via the example of the subtraction game.
2. Show via **backward induction** that player 1 in the subtraction game can guarantee payoff 1 via a **winning strategy**.
3. Define **subgame** and **subgame-perfect equilibrium**.
4. Recognize the notion of **extensive-form game** via the subtraction game example.
5. Calculate the winning strategy for each player in a finite game tree.
6. Use the the “line-item veto” and “mutually assured destruction” to describe and rationalize **winning strategies**.

This handout draws extensively from: Karlin, A. R., and; Peres, Y. (2016). Games in extensive form, Introduction. In *Game Theory, Alive* (pp. 114–118). AMS. and Nowak, M. A. (2006). *Evolutionary Dynamics: Exploring the equations of life*. Belknap Press of Harvard University Press.