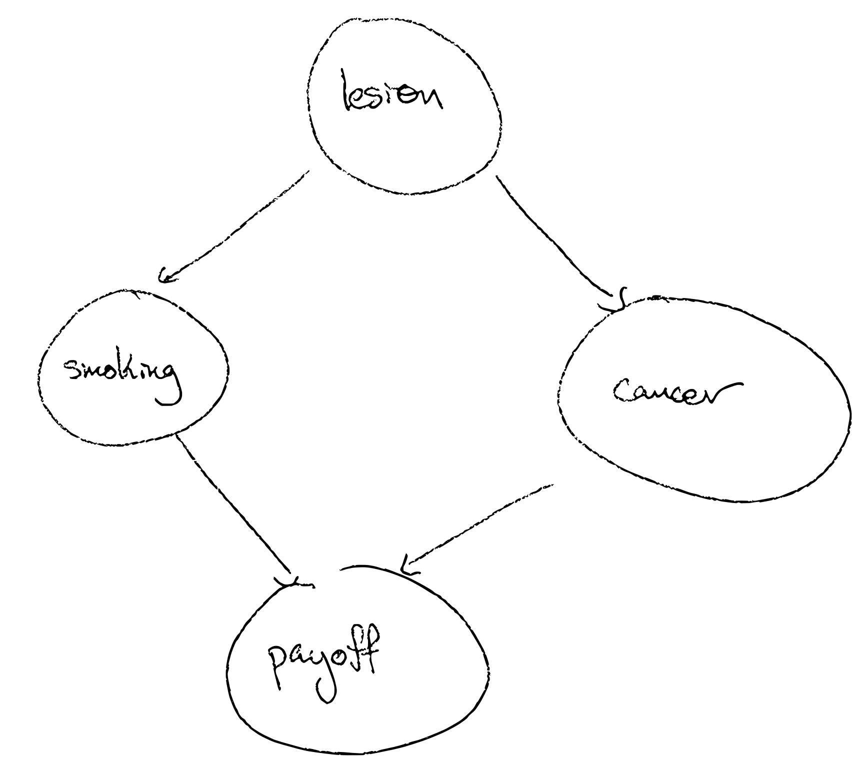
Recall (Smoking lesion)



Our goal is to make precise what graphs like these are and how to use them. Two key questions:

Q1. If we learn "smoking" = true, how do we update our belief about the value of "lesion"? (Bayes' theorem)

Q2. What does it mean for "smoking" and "cancer" to be "independent aside from the influence of 'lesion'"? (conditional independence).

Some more basic notions in probability theory

Let A, B be events.

Notation

(1) $A \wedge B$ ("A and B") is the event where both A and B occur.

(2) $\neg A$ ("not A") is the event that A does not occur.

(3) Let $P(A,B) := P(A \wedge B)$ be the probability that A and B both occur (according to the probability distribution P).

Fact Let $B_1, \ldots, B_n$ be a set of exhaustive and mutually exclusive events. Then $\sum_{i=1}^{n} P(B_i) = 1$ and

$$\sum_{i=1}^{n} P(A, B_i) = P(A).$$

Defn The conditional probability of A given B is

$$P(A|B) := \frac{P(A,B)}{P(B)}.$$

Ex. Let A = "it is rainy" B = "the ground is wet". Suppose that whenever it rains the ground is wet, and if it's not rainy, the ground is wet only 1/4 of the time. Suppose that it's rainy on half of days. Then P looks like

		B: wet	B: dry
A	rainy	$\frac{1}{2}$	0
	sunny	$\frac{1}{8}$	$\frac{3}{8}$

$$P(\text{wet} \mid \text{rainy}) = \frac{P(\text{wet and rainy})}{P(\text{rainy})} = \frac{1/2}{1/2} = 1$$

$$P(\text{wet} \mid \text{sunny}) = \frac{P(\text{wet and sunny})}{P(\text{sunny})} = \frac{1/8}{\frac{1}{2}} = \frac{1}{4}.$$

$$P(\text{rainy} \mid \text{wet}) = \frac{P(\text{rainy and wet})}{P(\text{wet})} = \frac{1/2}{1/2 + 1/8} = \frac{4}{5}.$$

This last example was interesting: even though I only specified $P(\text{wet} \mid \text{rainy})$, $P(\text{wet} \mid \text{sunny})$ and $P(\text{rainy})$ in the set-up, we were able to compute $P(\text{rainy} \mid \text{wet})$. In general, we have the following:

Thm (Bayes' rule)

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}.$$

Pf

$$P(B)P(A|B) = P(A,B) = P(A)P(B|A). \quad \blacksquare$$