

Lecture 20

Math S-Ar

07/23/2020

Plan for Today

- Sum of angles and double angles.

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- Presentations.

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- Composition of functions.

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Trigonometric Identities

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- Find the distance PQ using the distance formula.

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$$D = \sqrt{(\cos \beta - \cos \alpha)^2 + (-\sin \beta - \sin \alpha)^2}$$

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$$D = \sqrt{(\cos \beta - \cos \alpha)^2 + (-\sin \beta - \sin \alpha)^2}$$
- Find the distance PQ using the Law of Cosines.
$$D = \sqrt{2 - 2 \cos(\alpha + \beta)}.$$

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Addition Formulas

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Setting $\alpha = \beta$ in the above two statements, we get

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Double Angle Formulas

- $\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha).$

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Double Angle Formulas

- $\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha).$
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Solve $\sin 2t = \sqrt{2} \sin(t + \pi/2)$ for $0 \leq t < 2\pi$.

Use the double-angle formula for sine to replace $\sin 2t$ by $2 \sin t \cos t$. We also replace $\sin(t + \pi/2)$ by $\cos t$,

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The solutions of $\cos t = 0$ are $t = \pi/2$ and $t = 3\pi/2$.

The solutions to $2 \sin t - \sqrt{2} = 0$ are $t = \pi/4$ and $t = 3\pi/4$.

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Presentation Time!

Composition of Functions

The basics

- If f and g are some two functions, how exactly would we compute $f(g(2))$? Please explain in plain English.

Composition of Functions

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Put 2 into g ,

Composition of Functions

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