

# Local Maximum/Minimum

## Definition (5.7)

Let  $f$  be a real function defined on a metric space  $X$ . We say that  $f$  has a *local maximum* at a point  $p \in X$  if there is a  $\delta > 0$  such that  $f(q) \leq f(p)$  for all  $q \in X$  with  $d(p, q) < \delta$ .

*Local minimum* are defined likewise.

## Theorem (5.8)

Let  $f$  be defined on  $[a, b]$ . If  $f$  has a local maximum at a point  $x \in (a, b)$  and if  $f'(x)$  exists, then  $f'(x) = 0$ .

Then analogous statement for local minimum also holds.

# Generalized Mean Value Theorem

## Theorem (5.9)

*If  $f$  and  $g$  are continuous real functions on  $[a, b]$  which are differentiable in  $(a, b)$  then there is a point  $x \in (a, b)$  at which*

$$[f(b) - f(a)]g'(x) = [g(b) - g(a)]f'(x)$$

*Note that differentiability is not required at the endpoints.*

This theorem is often called a *generalized mean value theorem*

# Mean Value Theorem

## Theorem (5.10)

*If  $f$  is a continuous real function on  $[a, b]$  which are differentiable in  $(a, b)$  then there is a point  $x \in (a, b)$  at which*

$$[f(b) - f(a)] = [b - a]f'(x)$$

# Properties of Derivatives

## Theorem (5.11)

Suppose  $f$  is differentiable in  $(a, b)$

- (a) If  $f'(x) \geq 0$  for all  $x \in (a, b)$  then  $f$  is monotonically increasing.
- (b) If  $f'(x) = 0$  for all  $x \in (a, b)$  then  $f$  is constant.
- (c) If  $f'(x) \leq 0$  for all  $x \in (a, b)$  then  $f$  is monotonically decreasing.

# Continuity of Derivatives

## Theorem (5.12)

*Suppose  $f$  is a real differentiable function on  $[a, b]$  and suppose  $f'(a) < \gamma < f'(b)$ . Then there is a point  $x \in (a, b)$  such that  $f'(x) = \gamma$*

A similar result holds if  $f'(a) > f'(b)$

## Corollary

*If  $f$  is differentiable on  $[a, b]$  then  $f'$  cannot have any simple discontinuities on  $[a, b]$*