Neighborhood of Infinity

Definition

For any real c, the set of real numbers such that $x > c$ is called a neighborhood of $+\infty$ and is written $(c, +\infty)$.

Similarly the set of real numbers such that $x < c$ is called a neighborhood of $-\infty$ and is written $(c, -\infty)$.

Limits at Infinity

Definition

Let f be a real function defined on $E \subseteq \mathbb{R}$. We say

$$f(t) \to A \text{ as } t \to x$$

where A and x are in the extended real number system, if for every neighborhood U of A there is a neighborhood V of x such that $V \cap E$ is non empty and such that $f(t) \in U$ for all $t \in V \cap E, t \neq x$

It is clear that this definition coincides with the definition of limit on $\mathbb{R}$.

Arithmetic Operations

Theorem

Let f and g be defined on $E \subseteq \mathbb{R}$. Suppose

$$f(t) \to A \;\; g(t) \to B \text{ as } t \to x$$

Then

(a) $f(t) \to A'$ *implies* $A' = A$

(b) $(f+g)(t) \to A + B$

(c) $(fg)(t) \to AB$

(d) $(f/g)(t) \to A/B$

provided the right members of (b), (c) and (d) are defined.

Note that $\infty - \infty, 0 \cdot \infty, \infty/\infty, A/0$ are not defined.