# Partial Sums Formula

**Theorem**

*Given two sequences* $\{a_n\}$, $\{b_n\}$ *put*

$$A_n = \sum_{k=0}^{n} a_k$$

*if* $n \geq 0$; *put* $A_{-1} = 0$. *Then if* $0 \leq p \leq q$, *we have*

$$\sum_{n=p}^{q} a_n b_n = \sum_{n=p}^{q-1} A_n (b_n - b_{n+1}) + A_q b_q - A_{p-1} b_p$$

# Bounded Sequence Formula

**Theorem**

*Suppose*

(a) *The partial sums* $A_n$ *of* $\sum a_n$ *for a bounded sequence*

(b) $b_0 \geq b_1 \geq b_2 \geq \cdots$

(c) $\lim_{n\to\infty} b_n = 0$

*then* $\sum a_n b_n$ *converges.*

# Bounded Sequence Formula

**Theorem**

*Suppose*

(a) $|c_1| \geq |c_2| \geq |c_3| \geq \cdots$

(b) $c_{2m-1} \geq 0,\ c_{2m} \leq 0$

(c) $\lim_{n\to\infty} c_n = 0$

*then* $\sum c_n$ *converges.*

Series which satisfy $(b)$ are called *alternating series*. The theorem was known to Leibnitz.

# Radium of Convergence 1

## Theorem

*Suppose the radius of convergence of* $\sum c_n z^n$ *is* 1 *and suppose* $c_0 \geq c_1 \geq c_2 \geq \cdots$, $\lim_{n\to\infty} c_n = 0$. *Then* $\sum c_n z^n$ *converges at every point on the circle* $|z| = 1$, *except possibly at* $z = 1$.

# Absolute Convergence

The series $\sum a_n$ is said to *converge absolutely* if the series $\sum |a_n|$ converges.

### Theorem

*If $\sum a_n$ converges absolutely then $\sum a_n$ converges.*

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Notice if all terms are positive then absolute convergence is the same as convergence.

## Definition

If $\sum a_n$ converges but $\sum |a_n|$ diverges then we say that $\sum a_n$ converges *non-absolutely*.

For example $\sum \frac{(-1)^n}{n}$ converges conditionally.