Differentiation and Integration are Inverses

Theorem (6.20)

Let $f \in \mathcal{R}$ on $[a, b]$. For $a \leq x \leq b$ put

$$F(x) = \int_a^x f(t)dt$$

Then F is continuous on $[a, b]$; furthermore, if f is continuous at a point x_0 of $[a, b]$, then F is differentiable at x_0, and

$$F'(x_0) = f(x_0)$$

Fundamental Theorem of Calculus

Theorem (6.21)

If $f \in \mathcal{R}$ on $[a, b]$ and if there is a differentiable function F on $[a, b]$ such that $F' = f$, then

$$\int_a^b f(x \; dx = F(b) - F(a)$$

Integration by Parts

Theorem (6.22)

Suppose F ad G are differentiable functions on $[a, b]$, $F' = f \in \mathcal{R}$ and $G' = g \in \mathcal{R}$. Then

$$\int_a^b F(x)g(x)\ dx = F(b)G(b) - F(a)G(a) - \int_a^b f(x)G(x)\ dx$$