

1. A - It is possible to place a linear order on C so that we do NOT get an ordered field, and impossible to place a linear order of C so that we DO get an ordered field.

2. D - $|x + y| = |x| + |y|$ doesn't have to happen. For example, let $y = -x \neq 0$.

3. Yes, there is a metric space X with open sets U_i such that $\bigcap U_i$ is open. For example, if X is any set and d is the metric $d(x, x) = 0$ and $d(x, y) = 1$ for $x \neq y$, then this works.

4. No, not every bounded closed subset of a metric space is compact. If the metric space is \mathbb{R}^n with its usual metric, then yes. However, $\mathbb{Q} \cap [\sqrt{2}, \sqrt{3}]$ is closed and bounded in \mathbb{Q} , but not compact.

5. Yes, if $s_n \rightarrow s$ and $t_n \rightarrow t$, then $s_n t_n \rightarrow st$.

6. No, every bounded sequence of real numbers has a convergent subsequence. (In any bounded set, every sequence has a convergent subsequence. Whether or not the subsequence converges to something in the set is another issue).

7. A- Every compact space is complete (this is a theorem in Rudin), but not every complete space is compact (for example, \mathbb{R})

8. C- Intervals of convergence for power series are always either a bounded connected interval or infinite on both sides. (This is why the idea of "Radius" of convergence makes sense)

9. Yes, if a series converges but NOT absolutely, then there is a way to rearranging the terms of the sum to make the sum converge to whatever you desire.

10. Yes, there exists X, Y and $f : X \rightarrow Y$ with X closed and bounded by $f(X)$ NOT closed and bounded. For example, if $X = \mathbb{Q} \cap [\sqrt{2}, \sqrt{3}]$, then X is closed and bounded. If we let $Y = \mathbb{R}$ and $f : X \rightarrow Y$ be given as $f(x) = x$, then we find that the image is NOT closed.

11. Yes, a continuous function defined on a compact set must be uniformly continuous.

12. Yes, the composition of differentiable functions is always differentiable.

13. No, if you pick $f(x) = -|x|$, then f has a maximum at $x = 0$, but $f'(0) \neq 0$ since $f'(0)$ doesn't exist.

14. No, while this looks like L'hospital's rule, we must have that $\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} g(x) = 0$ or ∞ for it to apply. For an explicit counterexample, let $f(x) = x + 1$, $g(x) = x$ and $c = 1$. Then $\lim_{x \rightarrow c} f'(x)/g'(x) = 1$ while $\lim_{x \rightarrow 0} f(x)/g(x) = 2$.

15. No, if α isn't differentiable, then of course $\int_a^b f d\alpha$ can't equal $\int_a^b f \alpha' dx$.

16. No, let $[a, b] = [0, 1]$ and let $f_n(x) = x^n$. Then each f_n is continuous. However, $\lim_{n \rightarrow \infty} f_n = f(x)$ where $f(x) = \begin{cases} 0 & x \neq 0 \\ 1 & x = 0 \end{cases}$, which is clearly not continuous.

17. No, if you let $A = (0, 1) \cup (1, 2)$, then $\bar{A} = [0, 2]$ and it's interior is $(0, 2)$, which is NOT equal to A .

18. No, every continuous map from \mathbb{R} to \mathbb{Q} is constant. This follows since \mathbb{R} is connected, so $f(\mathbb{R})$ is connected. But the only connected subsets of \mathbb{Q} are points. To see this, let $A \subseteq \mathbb{Q}$ have two distinct points, say $p, q \in A$. Choose an irrational number x between p and q . Then $(-\infty, x)$ and (x, ∞) separate A .

19. Yes, If A is connected, \bar{A} is as well. For, if U and V disconnect \bar{A} , then U and V disconnect A , unless $A \subseteq U$ or $A \subseteq V$. Assume without loss of generality that $A \subseteq U$. Then $\bar{A} \subseteq \bar{U}$. But $\bar{U} \cap V = \emptyset$ so that $\bar{A} \cap V = \emptyset$, giving a contradiction.

20. No, there are functions $f : (0, 1) \rightarrow (0, 1)$ without fixed points, for example $f(x) = x/2$.