Math 170 Final Formula Sheet

December 6, 2008

1 Mathematical Models

Let $M_{n+1} = f(M_n)$ be a mathematical model (without the starting value specified).

Definition 1.0.1. M is a *equilibrium* if $M = f(M)$. An equilibrium M is a *stable equilibrium* if, whenever we let $M_0 = M \pm \epsilon$ (where ϵ is small) then the limit of M_n as $n \to \infty$ is M. Otherwise M is unstable.

2 Mandelbrot and Julia Sets

Definition 2.0.2. The *Mandelbrot set* is the collection of complex numbers c such that the mathematical model

$$M^c_{n+1} = (M^c_n)^2 + c, \qquad M_0 = 0$$

does not go to infinity as n goes to infinity.

Definition 2.0.3. The *Julia set at d* is the collection of complex numbers c such that the mathematical model

$$J^d_{n+1} = (J^d_n)^2 + d, \qquad J_0 = c$$

does not go to infinity as n goes to infinity.

3 Complex Numbers

Definition 3.0.4. An *imaginary number* is one of the form ai where a is real and $i = \sqrt{-1}$

Definition 3.0.5. A *complex number* is one of the form $a+bi$ where a, b are real and $i = \sqrt{-1}$

Theorem 3.0.6. *For all complex numbers,* $(a+bi)+(c+di) = (a+c)+(b+d)i$

Theorem 3.0.7. *For all complex numbers,* $(a+bi) \times (c+di) = (ac-bd)+(ad+bc)i$

Theorem 3.0.8. *Let* $a+bi$ *be a complex number. The complex conjugate of* $a+bi$ $(\overline{a+bi})$ *is* $a-bi$*. Then* $(a+bi)(a-bi) = a^2+b^2$

Theorem 3.0.9 (Fundamental Theorem of Algebra)**.** *For every polynomial* $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ *with each* a_i *a complex number, there is a complex number* α *such that* $f(\alpha) = 0$

Theorem 3.0.10 (Quadratic Equation)**.** *If* $ax^2+bx+c = 0$ *then*

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

4 Change of Base

Definition 4.0.11. If $a_n, \ldots, a_0, a_{-1}, \ldots, a_{-k}$ are greater than or equal to 0 and less than m then $a_n a_{n-1} \ldots a_0$ base $m = a_n * m^n + a_n * m^{n-1} + \cdots a_1 * m + a_0 + a_{-1}m^{-1} + \cdots + a_{-k}m^{-k}$