

Math 104-006

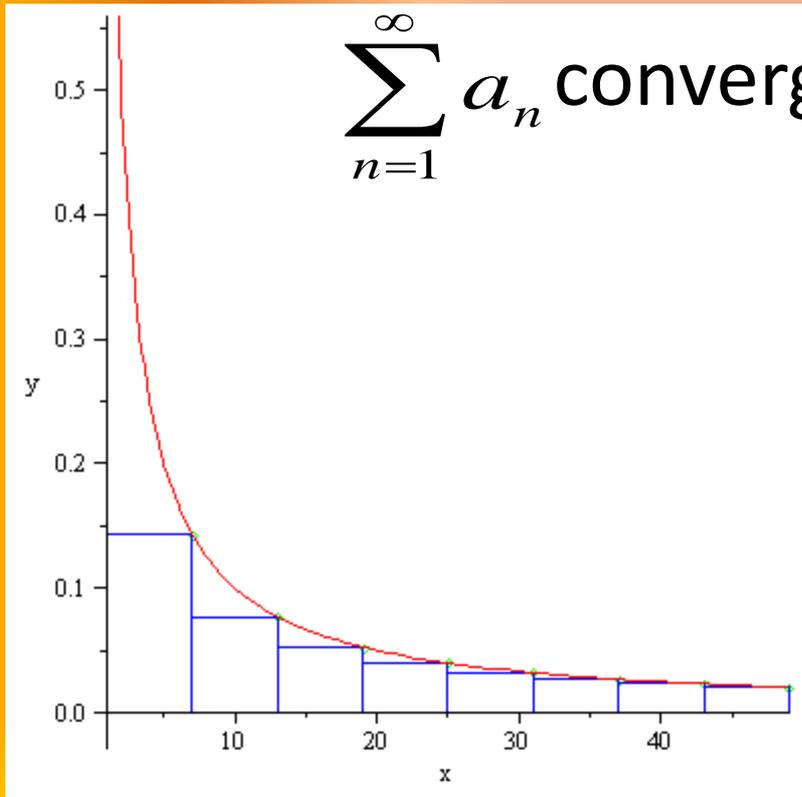
Chapter 12.3: The Integral Test and Estimates of Sums

Outline For Today

- Integral Test
- Bounds on the Error of the Integral Test

Integral Test

- If $f(x)$ is a continuous, positive, decreasing function on $[1, \infty)$ and $a_n = f(n)$ then



$$\sum_{n=1}^{\infty} a_n$$

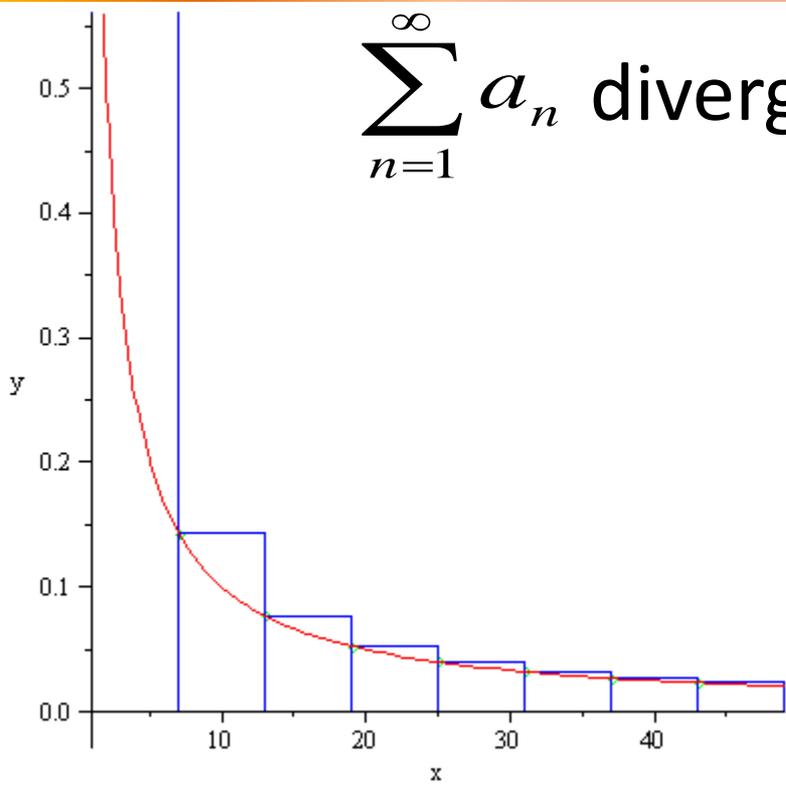
converges if $\int_1^{\infty} f(x) dx$ converges

Note we do not need
to start at $n = 1$

Integral Test Continued

- If $f(x)$ is a continuous, positive, decreasing function on $[1, \infty)$ and $a_n = f(n)$ then

$\sum_{n=1}^{\infty} a_n$ diverges if $\int_1^{\infty} f(x) dx$ diverges



Note we do not need to start at $n = 1$

Integral Test Continued

This does **NOT** mean $\sum_{n=1}^{\infty} a_n = \int_1^{\infty} f(x) dx$

p-series

We call the sum $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots$

a p-series

For what values p does $\sum 1/n^p$ converge?

A) $p > 0$

D) $p \leq 1$

B) $p > 1$

E) all p

C) $p \geq 1$

F) None of the above

For what values p does $\sum 1/n^p$ converge?

A) $p > 0$

B) $p > 1$

C) $p \geq 1$

D) $p \leq 1$

E) all p

F) None of the above

Example

Does $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ converge?

We want to do a comparison test with $\int_1^{\infty} \frac{\ln(x)}{x} dx$

But first we need to check that $\frac{\ln(x)}{x}$ is continuous and decreasing.

Example Continued

It is clear that $\frac{\ln(x)}{x}$ is continuous so we just need to check that it is decreasing.

$$\text{We see that } \frac{d}{dx} \left(\frac{\ln(x)}{x} \right) = \frac{(1/x)x - \ln(x)}{x^2} = \frac{1 - \ln(x)}{x^2}$$

So if $x > 1$ $\frac{d}{dx} \left(\frac{\ln(x)}{x} \right) < 0$, and $\frac{\ln(x)}{x}$ is decreasing

Example Continued

So $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ converges if and only if $\int_1^{\infty} \frac{\ln(x)}{x} dx$

converges.

But if $u = \ln(x)$, $du = 1/x$ and $\int_1^{\infty} \frac{\ln(x)}{x} dx = \int_1^{\infty} \frac{1}{u} du$

Which diverges. So $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ diverges.

Error

Let a_n be a sequence, let $s_n = \sum_{i=1}^n a_i$ and let $s = \sum_{i=1}^{\infty} a_i$

Then $R_n = s - s_n$ is the remainder for the estimate of the sum using the n th partial sum.

If $f(x)$ is continuous, decreasing and $a_n = f(n)$ then

$$\int_{n+1}^{\infty} f(x) dx \leq R_n \leq \int_n^{\infty} f(x) dx$$

How many terms of $\sum e^{-n}$ must we add to so that the size of the remainder is less than 0.001?

A) $n = 5$

D) $n = 8$

B) $n = 6$

E) $n = 9$

C) $n = 7$

F) None of the above

How many terms of $\sum e^{-n}$ must we add to so that the size of the remainder is less than 0.001?

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