Math 104-006

Chapter 12.2: Series

Outline For Today

- Series
- Limits of Series
- Operations on Series

Infinite Series

- A series is what you get when you try and add up infinitely many numbers:

- $a_1 + a_2 + a_3 + \ldots = \sum_{n=0}^{\infty} a_n = \sum a_n$

Which we call an **infinite series**

Partial Sum and Infinite Series

- $a_1 + a_2 + a_3 + \ldots + a_n = s_n = \sum_{i=1}^{n} a_i$
- If $\lim_{n\to\infty} s_n$ exists we say $\sum_{n=1}^{\infty} a_n$ converges and

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n$$

- Otherwise we say $\sum_{n=1}^{\infty} a_n$ diverges

Geometric Series

- A geometric series is one of the form
- $a + ar + ar^2 + ar^3 + \ldots = \sum_{n=0}^{\infty} ar^n$
- If $|r| < 1$ then $\sum_{n=0}^{\infty} ar^n$ converges and is $\frac{a}{1-r}$
- If $|r| \geq 1$ then $\sum_{n=0}^{\infty} ar^n$ is divergent.

Try An Example

What is $4 + 2/3 + 1/9 + \ldots$?

A) 24/5

B) 5/24

C) 24

D) 12

E) 12/7

F) Divergent

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Another Example

What is 0.1212... as a fraction?

A) 5/9

B) 6/11

C) 4/33

D) 1/12

E) 2/9

F) 4/99

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Test for divergence

- If $\sum a_n$ is convergent then $\lim_{n\to\infty} a_n = 0$
- So if $\lim_{n\to\infty} a_n \neq 0$ or $\{a_n\}$ is divergent then
- So is $\sum a_n$

Harmonic Series

- The series $\sum \frac{1}{n}$ is divergent

Properties of Limits

If $\sum a_n$ and $\sum b_n$ are convergent series and

c is a constant then $\sum ca_n$, $\sum a_n + b_n$, and $\sum a_n - b_n$ are convergent with

$$\sum (a_n + b_n) = \sum a_n + \sum b_n$$

$$\sum (a_n - b_n) = \sum a_n - \sum b_n$$

$$\sum ca_n = c\sum a_n$$