Math 104-006

Chapter 6.5: Average Value of a Function

Outline For Today

- Average Values of Functions

Average Value of a Sequence of Numbers

- If we have numbers $a_1, a_2, \ldots, a_n$, their average:

$$\text{Average} = \frac{a_1 + a_2 + \ldots + a_n}{n}$$

Approximation to the Average Value of a Function

To approximate the average value f(x) on an interval (a, b) we can break the interval up into smaller intervals, pick a representative of the interval, and average the representatives.

Approximate Average Value= $$\frac{\sum_{i=1}^{n} f(x_i^*)}{n}$$

Approximation to the Average Value of a Function

But if Δx is the width of the interval then we have:

$$\Delta x = \frac{b-a}{n}$$

Or equivalently

$$n = \frac{b-a}{\Delta x}$$

Approximation to the Average Value of a Function

So the approximation to the average value of f(x) on the interval (a,b) is

Approximate Average Value

$$\frac{\sum_{i=1}^{n} f(x_i^*)}{n} = \frac{\sum_{i=1}^{n} f(x_i^*)}{\frac{b-a}{\Delta x}} = \frac{1}{b-a}\sum_{i=1}^{n} f(x_i^*)\Delta x$$

Actual Average Value

- As the number of intervals goes to infinity the error in the approximation goes to zero.

$$\text{Average Value} = \lim_{n\to\infty} \frac{1}{b-a} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

$$== \quad \frac{1}{b-a} \int_a^b f(x) \cdot dx$$

Your Turn

Suppose a stock has a value at time t of $\$t^2$.

What is the average value from time 0 to 12?

A) 48

B) 144

C) 12

D) 434

E) 18

F) 36

Question

Suppose a stock has a value at time t of $\$t^2$.

What is the average value from time 0 to 12?

A) 48

B) 144

C) 12

D) 434

E) 18

F) 36

Mean Value Theorem

If f(x) is a continuous function on [a, b] then there is a c in [a,b] such that

$$f(c) = f_{average} = \frac{1}{b-a}\int_a^b f(x)\cdot dx$$

Or equivalently

$$f(c)\cdot(b-a) = \int_a^b f(x)\cdot dx$$

Lets Try Another One

Suppose a stock has a value at time t of $\$t^2$ from time t=0 to t=12. At what time does it reach its average value?

A) $3\sqrt{3}$

B) $4\sqrt{3}$

C) $6\sqrt{2}$

D) 6

E) 4

F) $8\sqrt{2}$

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