MATH 104-006

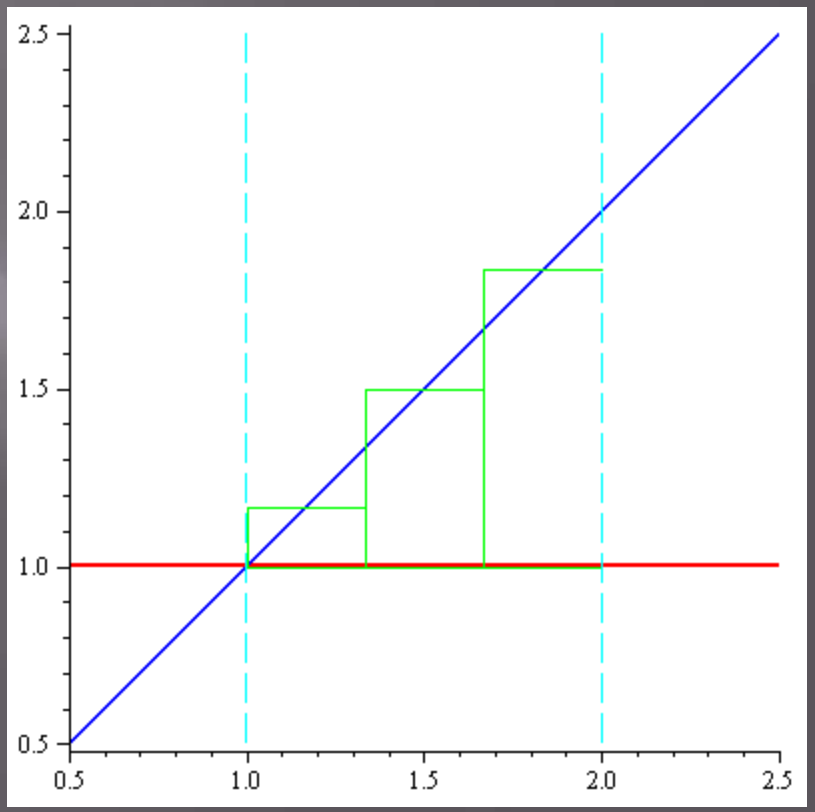
Chapter 6.1: Areas Between Curves

Outline For Today

- Approximating Areas Between Curves
- Area Between Curves as an Integral
- Area Between Curves Which Intersect

Approximate Area Between Curves

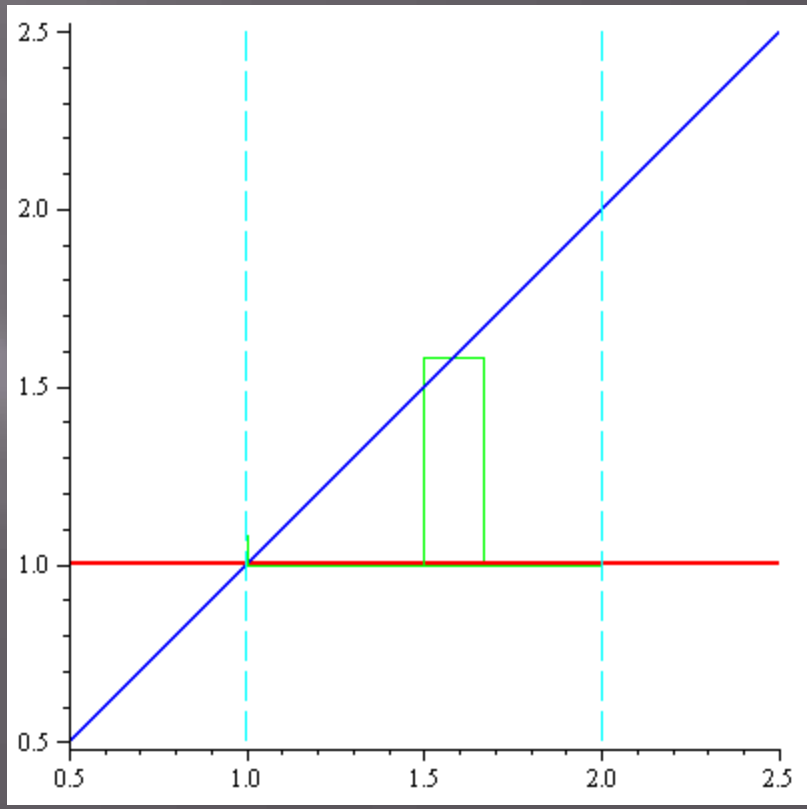
- We want to approximate the area between two curves by using boxes
  - The bottom of each box is on one curve and the top of the box is on the other curve



Area of One Box

- The area of one box is: $[f(x^*) - g(x^*)]\Delta x$

- Where Δx is the width of the box.
- f is the blue function
- g is the red function



Area of Approximation

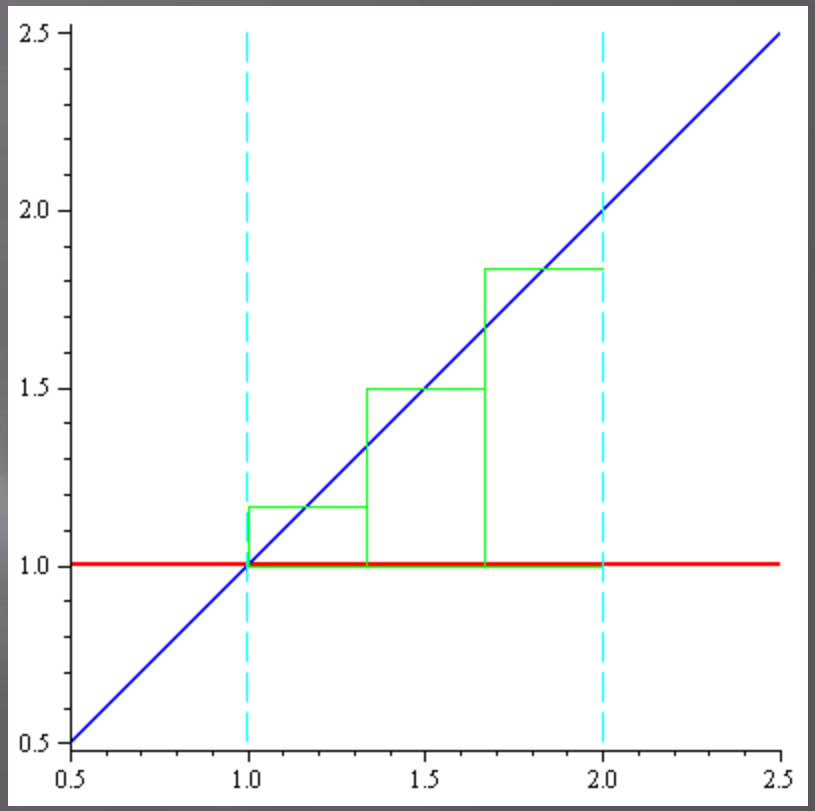
If f(x) is greater than or equal g(x) on the interval.

The area of the approximation is the sum of the area of all the boxes.

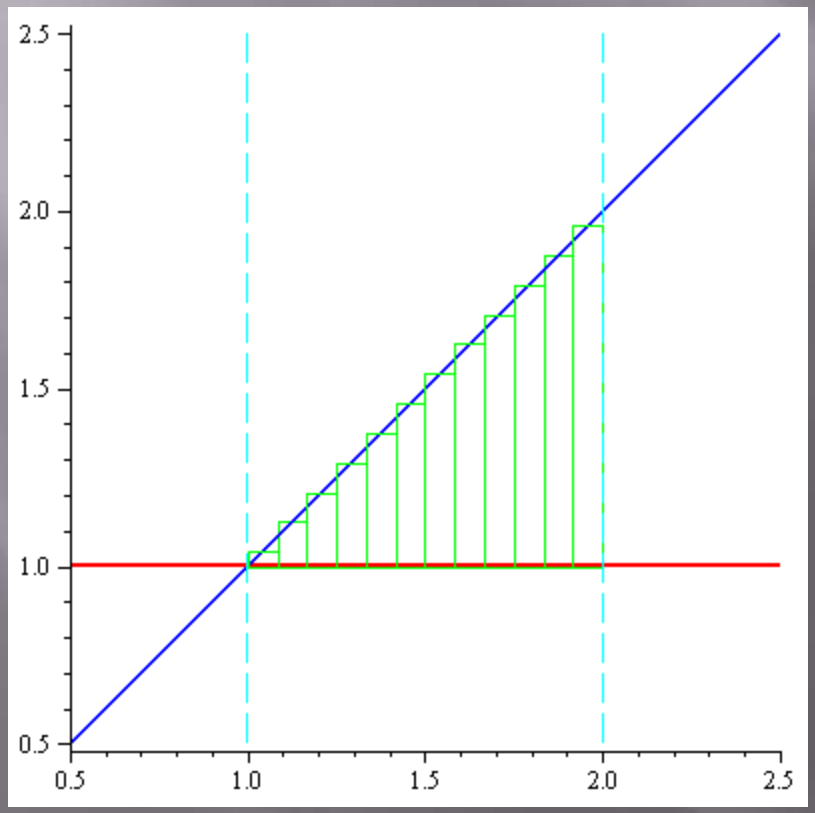
$$\sum_{i=0}^{n}[\,f(x_i^*)-g(x_i^*)]\Delta x$$

More Boxes

- The more boxes we use, the better the approximation



3 boxes



12 boxes

Actual Area

- As the number of boxes goes to infinity the error in the approximation goes to zero.

$$\text{Area} = \lim_{n\to\infty} \sum_{i=0}^{n} [\, f(x_i^*) - g(x_i^*)]\Delta x$$

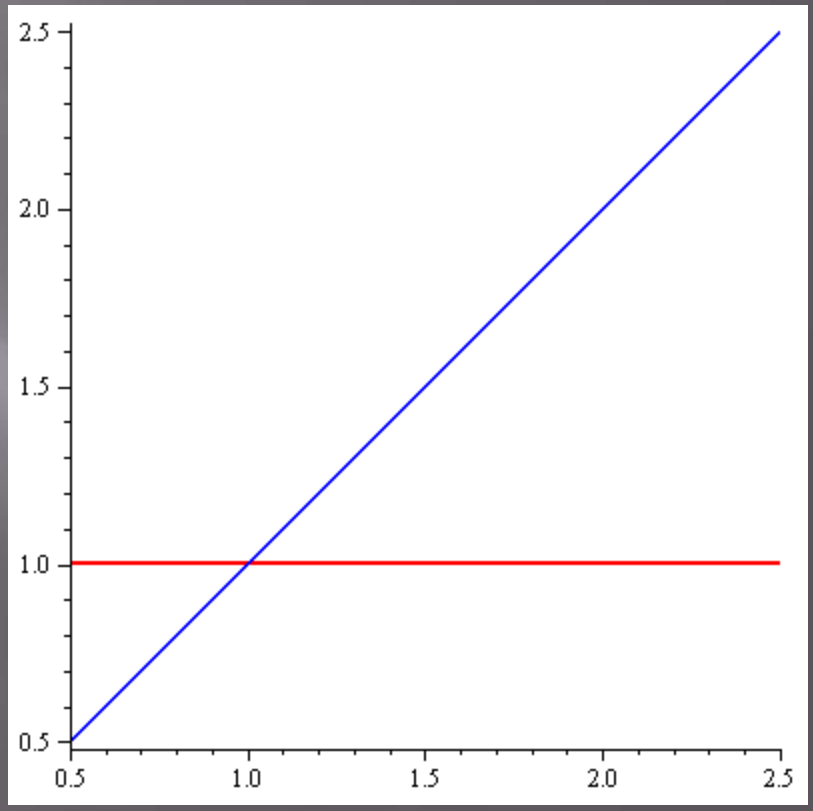
$$\text{Area} = \int_a^b [\, f(x) - g(x)]dx \qquad \text{if} \quad f(x) \geq g(x)$$

Area of Example

$$\text{Area} = \int_1^2 [x - 1]\,dx$$

$= x^2/2 - x$, from 1 to 2

$= (4\text{-}2) - (1/2 - 1) = 1/2$

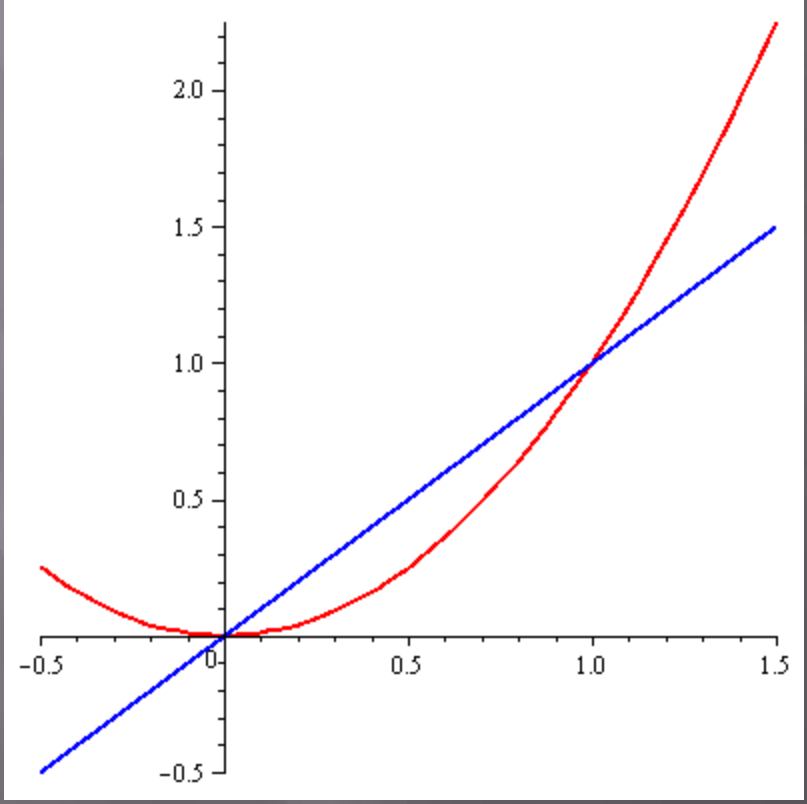


f(x) = x g(x) = 1

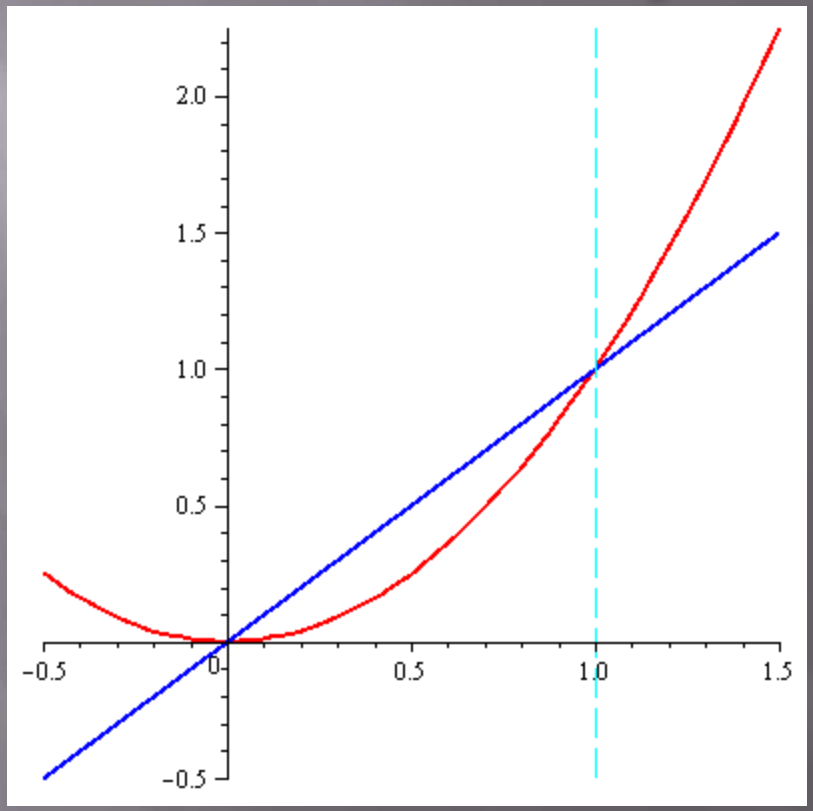
Area Enclosed By Curves

Lets find the area enclosed by

$$f(x) = x, g(x) = x^2$$



Area Enclosed By Curves



$$\int_0^1 (x^2 - x)dx = x^3/3 - x^2/2$$ for x = 0 to 1

$$= (1/3 - 1/2) - 0 = 1/6$$

Your Turn

What is the area enclosed by f(x) = sin(x) and g(x) = cos(x) restricted to the interval $(-\pi, \pi/2)$

A) $\pi/2$

B) π

C) 2

D) $2\sqrt{2}$

E) $1/\sqrt{2}$

F) $-\pi$

Answer

What is the area enclosed by f(x) = sin(x) and g(x) = cos(x) restricted to the interval $(-\pi, \pi/2)$

A) $\pi/2$

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D) $2\sqrt{2}$

E) $1/\sqrt{2}$

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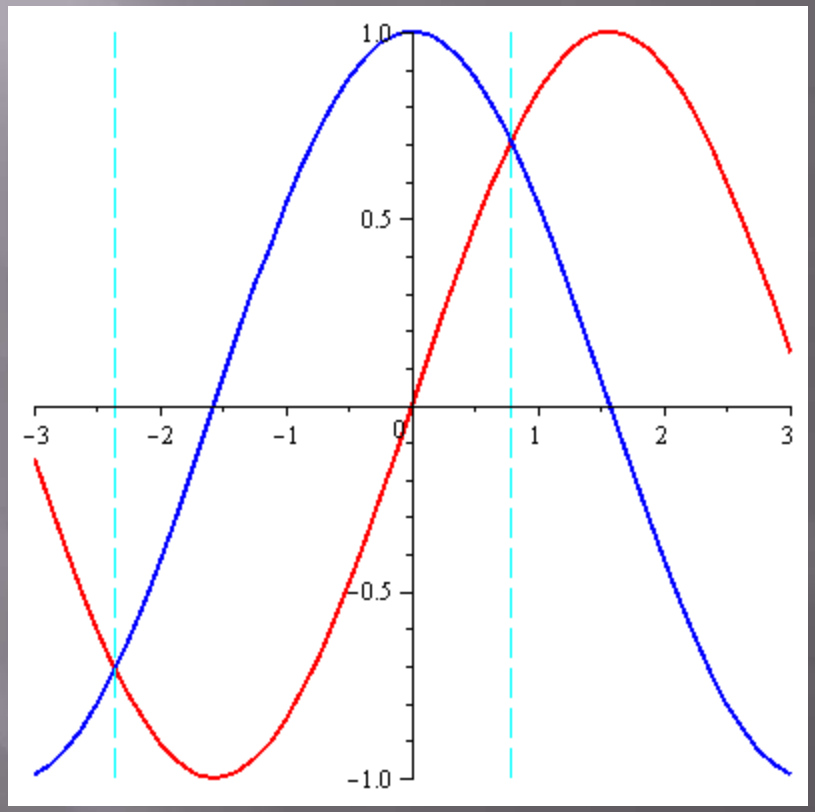
Lets Look At The Graph



Lets Try Another

What is the area between the curves $f(x) = x$ and $g(x) = 1$ on the interval $[0, 2]$?

A) 2

B) 1

C) 0

D) 1/2

E) 4

F) – 1

Answer

What is the area between the curves $f(x) = x$ and $g(x) = 1$ on the interval $[0, 2]$?

A) 2

B) 1

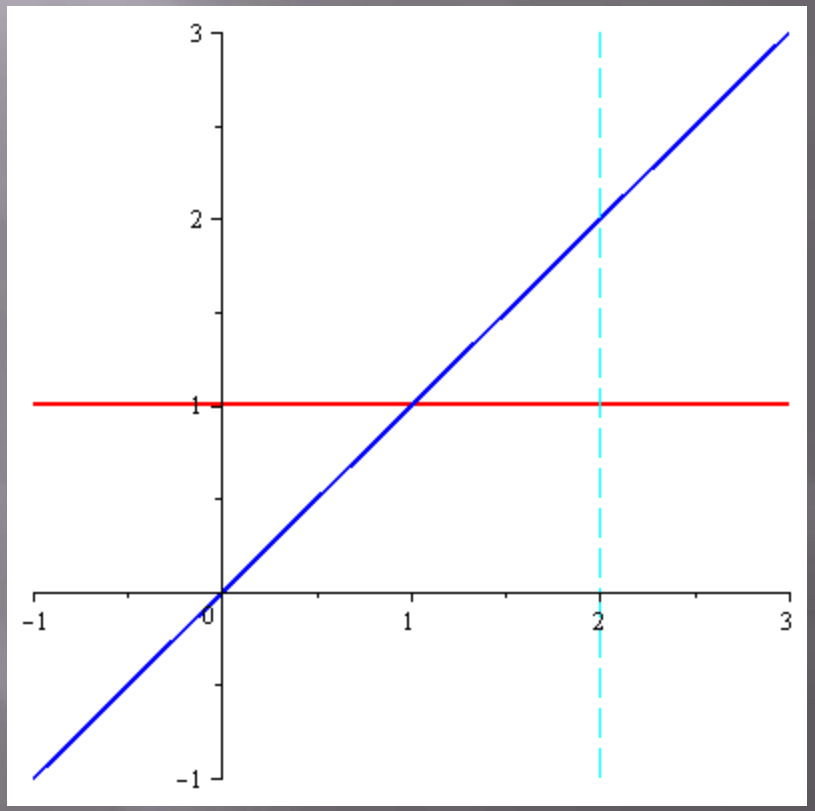
C) 0

D) 1/2

E) 4

F) – 1

Lets Look At The Graph



Formula For Area

Area between two curves given by f(x) and g(x) on the interval a to b is

$$\text{Area} = \int_a^b |\, f(x) - g(x) \,|\, dx$$

General Strategy for Finding Areas Between Curves

- 1) Find the left and right boundaries (if not given)
- 2) Find points of intersection of the two curves
- 3) Between any two points of intersection integrate the top curve and subtract from it the integral of the bottom curve
- 4) Add up all of the areas in step 3.

Find The Area Between $y = x-1$ and $y^2 = 2x + 6$

A) 12

B) 1

C) 0

D) 18

E) 6

F) 2

Answer

Find The Area Between $y = x-1$ and

$y^2 = 2x + 6$

A) 12

B) 1

C) 0

D) 18

E) 6

F) 2