Math 104-006

Chapter 12.5: Alternating Series

Outline For Today

- Alternating Series
- Alternating Series Tests

Alternating Series

- An alternating series is one where the terms alternate between being positive and negative.

For example

$1 - 1/2 + 1/4 - 1/8 + \ldots.$

Alternating Series Continued

- Formally, a series $\sum_{n=0}^{\infty} a_n$ is alternating if there

is a sequence $\{b_n\}_{n=0}^{\infty}$ such that

$b_n \geq 0$ for all n and

$a_n = (-1)^n b_n$

Alternating Series Test

- Suppose $\sum_{n=0}^{\infty}(-1)^n b_n$ is an alternating series

Such that

(i) $b_{n+1} \leq b_n$ for all n.

(ii) $\lim_{n\to\infty} b_n = 0$

Then the series is convergent.

Example

- Does $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n}$ converge?

- We see that 1/n > 1/(n+1) and

$$\lim_{n\to\infty} \frac{1}{n} = 0$$

- So by the alternating series test

$\sum_{n=1}^{\infty}(-1)^n \frac{1}{n}$ converges

Alternating Series Error Theorem

If $\sum_{n=0}^{\infty}(-1)^n b_n$ is the sum of an alternating series

such that

(i) $0 \le b_{n+1} \le b_n$ for all n.

(ii) $\lim_{n\to\infty} b_n = 0$

then $|R_n| = |s - s_n| \le b_{n+1}$

Example

- How many terms of $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2}$ do we need to add to ensure that the size of the remainder is less than 1/101?

A) 10

B) 11

C) 100

D) 9

E) 101

F) None of the above

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